# 30 years of Bucharest-Stockholm collaboration in the field of emission processes

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# Our publications (11.08.2019)





## We investigated cold emission processes

defined by the splitting of a nucleus into two or more fragments in their ground or low-lying states which energetically are more bound

 Proton and two-proton emission (predicted in 1960 by V.I. Goldansky)
 Alpha decay (discovered in 1899 by E. Rutherford)
 Heavy-cluster emission (predicted in 1977 by A. Sandulescu)

## **1. Proton emission**

PRL 96, 072501 (2006)

PHYSICAL REVIEW LETTERS

week ending 24 FEBRUARY 2006

#### Systematics of Proton Emission

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A very simple formula is presented that relates the logarithm of the half-life, corrected by the centrifugal barrier, with the Coulomb parameter in proton decay processes. The corresponding experimental data lie on two straight lines which appear as a result of a sudden change in the nuclear shape marking two regions of deformation independently of the angular momentum of the outgoing proton. This feature provides a powerful tool to assign experimentally quantum numbers in proton emitters.

DOI: 10.1103/PhysRevLett.96.072501

PACS numbers: 21.10.Tg, 23.50.+z, 24.10.Eq

## **Review paper on proton emission**



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## Theories of proton emission

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> Accepted 2 November 2005 Available online 4 January 2006 editor: G.E. Brown

## Proton emission systematics for reduced (on angular momentum) half life



FIG. 1. (a) Logarithm of the experimental half-lives corre sponding to proton decay as a function of the Q value. The data are taken from Ref. [4]. (b) Values of  $\log_{10}T_{red}$ , Eq. (6), as a function of the Coulomb parameter  $\chi$ . The numbers labeling the different symbols correspond to the l values of the outgoing proton. The two lines are computed according to Eq. (8).

$$T_{\rm red} = \frac{T_{1/2}}{C_l^2}$$

$$C_l(\chi, \rho) = \exp\left[\frac{l(l+1)}{\chi} \tan\alpha\right],$$
$$\cos^2 \alpha = \frac{Q_p}{V_c(R)} = \frac{\rho}{\chi},$$

$$og_{10}T_{red}^{(k)} = a_k(\chi - 20) + b_k,$$
  

$$a_1 = 1.31, \qquad b_1 = -2.44 \quad \text{for } Z < 68$$
  
and 
$$a_2 = 1.25, \qquad b_2 = -4.71 \quad \text{for } Z > 68, \qquad (8)$$

# Particle dynamics in the external barrier explains the dependence on Coulomb parameter and proton angular momentum



 $f_{lj}(\chi,\rho) = N_l H_l^{(+)}(\chi,\rho)] \equiv N_l C_l H_0^{(+)}(\chi,\rho)$ 

## The two lines are explained by

PHYSICAL REVIEW C 80, 024310 (2009)

#### Universal decay rule for reduced widths

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Emission processes including  $\alpha$  decay, heavy cluster decay, and proton and di-proton emission are analyzed in terms of the well-known factorization between the penetrability and the reduced width. By using a shifted harmonic oscillator plus Coulomb cluster-daughter interaction it is possible to derive a linear relation between the logarithm of the reduced width squared and the fragmentation potential, defined as the difference between the Coulomb barrier and the Q value. This relation is fulfilled with a good accuracy for transitions between ground states, as well as for most  $\alpha$  decays to low-lying 2<sup>+</sup> excited states. The well-known Viola-Seaborg rule, connecting half-lives with the Coulomb parameter and the product between fragment charge numbers, as well as the Blendowske scaling rule, connecting the spectroscopic factor with the mass number of the emitted cluster, can be easily understood in terms of the fragmentation potential. It is shown that the recently evidenced two regions in the dependence of reduced proton half-lives versus the Coulomb parameter are directly connected with the corresponding regions of the fragmentation potential.

DOI: 10.1103/PhysRevC.80.024310

PACS number(s): 21.10.Tg, 23.50.+z, 23.60.+e, 23.70.+j

## By using a schematic ho + Coulomb interaction one obtains for the reduced width $\gamma^2$



in terms of fragmentation potential  $V_{\text{frag}}(r_B) = V_C(r_B) - Q$ . and cluster amplitude  $A_0$ 

# The two lines in systematics correspond to two different regions of the fragmentation potential V<sub>c</sub>-Q (b)



# By crossing Z=68 one has a large shape transition from $\beta \sim 0.3$ up tp $\beta \sim -0.2$



Universal law for reduced widths is also fulfilled by alpha emitters below and above <sup>208</sup>Pb D.S. Delion, A. Dumitrescu (At.Data Nucl.Data Tab, in prep.)



Alpha-decay log(width)-log(penetrability) dependence by using the fit parameters is equivalent to the Viola-Seaborg rule



Universal law for reduced widths is valid for all emission processes: proton, alpha & cluster decays



# One obtains a general log(width)-log(penetrability) dependence for all emission processes by using the corresponding fit parameters



## An equivalent explanation is given by

PHYSICAL REVIEW C 85, 011303(R) (2012)

#### Effects of formation properties in one-proton radioactivity

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It is shown that the proton formation probability, extracted from experimental data corresponding to one-proton radioactivity, is divided into two regions when plotted as a function of an universal parameter. This parameter is derived from a microscopic description of the decay process. In this way we explain the systematics of proton emission half-lives. At the same time the formation probability is shown to be a useful quantity to determine the deformation property of the mother nucleus.

DOI: 10.1103/PhysRevC.85.011303

PACS number(s): 23.50.+z, 21.30.Fe, 21.60.Gx, 21.10.Tg



FIG. 2. (Color online) Proton-decay formation amplitudes  $\log_{10} |RF(R)|^2$  extracted from experimental data as a function of  $\rho'$ . Squares correspond to nuclei with N < 75 ( $Z \le 67$ ) while circles are for  $N \ge 75$  (Z > 67).

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_l} = \frac{\ln 2}{\nu} \left| \frac{H_l^+(\chi, \rho)}{R \mathcal{F}_l(R)} \right|^2,$$

$$\rho' = \sqrt{AZ_p Z_d (A_d^{1/3} + A_p^{1/3})}, A = A_d A_p / (A_d + A_p),$$

## Two proton emission from superfluid emitters

PHYSICAL REVIEW C 87, 034328 (2013)

#### Simple approach to two-proton emission

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The two-proton decay process is studied by using a simple approach within the framework of scattering theory. We assume that the decaying nucleus is in a pairing state and, therefore, the two-particle wave function on the nuclear surface corresponds to the two protons moving in time-reversed states. This allows us to sustain a simplified version of the decay where the protons are simultaneously emitted with the same energies. We thus obtain a coupled system of radial equations with outgoing boundary conditions. We use similar proton-proton interactions to solve BCS equations and to describe external two-proton dynamics. A strong dependence of the pairing gap and decay width upon the proton-proton interaction strength is revealed. The experimental half-lives of <sup>45</sup>Fe and <sup>48</sup>Ni are reproduced by using a realistic proton-proton interaction.

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PACS number(s): 21.10.Tg, 23.50.+z, 25.70.Ef

## Pairing wave function is peaked on nuclear surface and around the angle $\phi \sim 45^{\circ}$ ( $r_1 \sim r_2$ )



FIG. 3. (a) Two-proton radial wave function in  ${}^{45}$ Fe, given by Eq. (2.7), as a function of the coordinate r for  $\varphi = 45^{\circ}$ . The parameters of the proton-proton interaction in Eq. (2.14) are  $v_0 = 35$  MeV and  $r_0 = 2$  fm. (b) Same as in panel (a) but as a function of the angle  $\varphi$  corresponding to the maximal value of the wave function at r = 6 fm.

$$r_{1} = r \cos \varphi, \quad r_{2} = r \sin \varphi,$$

$$r \in [0, \infty), \quad \varphi \in \left[0, \frac{\pi}{2}\right].$$

$$r_{1}$$

$$r_{1}$$

$$r_{2}$$

$$r_{2}$$

$$r_{2}$$



## pp interaction

$$v(r_{12}) = -v_0 e^{-(r_{12}/r_0)^2} + V_C(e^2, r_0, r_{12})$$

# We considered radial motion for a fixed angle $\varphi$ The main effect on the decay width is given for $\varphi \sim 45^{\circ}$ (r<sub>1</sub> ~ r<sub>2</sub>)



FIG. 6. The ratio of the angular-dependent width given by Eq. (2.31) and its value at  $\varphi = 45^{\circ}$  versus the angle  $\varphi$  for  $v_0 = 35 \text{ MeV}$  and  $r_0 = 2 \text{ fm}$ .

# Half life strongly depends on the pairing strength (and the pairing gap)

$$\underbrace{v(r_{12}) = -v_0 e^{-(r_{12}/r_0)^2} + V_C(e^2, r_0, r_{12})}_{\text{is } v_0 = 35 \text{ MeV}}$$



FIG. 4. Proton pairing gap versus proton-proton strength vo in 45Fe.



FIG. 7. Theoretical two-proton half-life for  ${}^{45}$ Fe, versus the matching proton radius for various proton-proton interaction strengths  $v_0$  in Eq. (2.14) (solid lines). The dashed lines indicate the upper and lower experimental limits.

## 2. Alpha decay Microscopic description in deformed nuclei in terms of the mean field + pairing approach

PHYSICAL REVIEW C

VOLUME 46, NUMBER 4

OCTOBER 1992

#### Alpha widths in deformed nuclei: Microscopic approach

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A microscopic approach to the alpha decay problem in deformed nuclei is presented. The nuclear wave functions are calculated in the frame of the Nilsson + BCS approximation, making use of a realistic deformed mean field. A large configuration space has been employed in the calculation of the formation amplitude while the penetration process has been treated within the WKB approximation. The calculated widths agree with the experimental data within a factor of about 3. Effects due to deformation are also discussed. Applications are presented for Ra, Rn, and Th isotopes.

PACS number(s): 23.60.+e

# Formation amplitude is the overlap between parent and daughter \* alpha wave functions

$$\mathscr{F}(\mathbf{R}_{\alpha}) = \langle \alpha D | P \rangle = \int d\mathbf{x}_{\alpha} d\mathbf{x}_{D} \left[ \psi_{\alpha}^{(\beta_{\alpha})}(\mathbf{x}_{\alpha}) \Psi^{(D)}(\mathbf{x}_{D}) \right]^{*} \Psi^{(P)}(\mathbf{x}_{P})$$

By using the cm and relative coordinates it becomes a superposition of ho orbitals depending on alpha-core radius with four times sp ho parameter 4β

$$\mathscr{F}_{\alpha}(\mathbf{R}) = \sum_{L_{\alpha}} \mathscr{F}_{L_{\alpha}}^{(\alpha)}(\mathbf{R}) = \sum_{L_{\alpha}} \sum_{N_{\alpha}} W(N_{\alpha}L_{\alpha}) \phi_{N_{\alpha}L_{\alpha}M_{\alpha}}^{(4\beta)}(\mathbf{R}).$$

where W-coefficients depend on Nilsson expansion coefficients and BCS amplitudes

## Formation probability is peaked on the nuclear surface but the total decay width is underestimated



FIG. 4. Square of the alpha-formation amplitude (in fm<sup>-3</sup>) vs R. The solid line corresponds to the inclusion of quadrupole deformation; the dashed line to the quadrupole + octupole case.

# Microscopic description of the decay width requires an increase of the radial tail

PHYSICAL REVIEW C

VOLUME 54, NUMBER 1

JULY 1996

#### New single particle basis for microscopic description of decay processes

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A single particle basis consisting of two different harmonic oscillator representations is introduced with the aim of studying microscopically  $\alpha$ - and cluster-decay processes. A correct description of the wave functions at large distances is obtained within a minimal single particle basis. Experimental data corresponding to a large number of  $\alpha$  decay transitions from even-even nuclei are well reproduced. [S0556-2813(96)01406-9]

PACS number(s): 21.60.Gx, 23.60.+e



## Mixed single particle basis with two ho parameters Alpha clustering is connected to the second part of the basis



$$\psi_{Elj\Omega}(\xi) = u_{Elj}(r) [i^l Y_l(\hat{r}) \chi_{1/2}(s)]_{j\Omega}$$

N. N.

$$u_{\alpha}(r) = \sum_{2n_1+l=N_1 \le N_0} c_{\alpha n_1}^{(1)} R_{n_1 l}^{(\lambda_1)}(r) + \sum_{2n_2+l=N_2 > N_0} c_{\alpha n_2}^{(2)} R_{n_2 l}^{(\lambda_2)}(r)$$

712 (X.)

 $f_k = \frac{\lambda_k}{\lambda_0}, \quad k = 1, 2.$ 

$$\lambda = f \lambda_0 = f \frac{M_0 \omega}{\hbar}$$

# Saturation property of the mixed sp basis versus the number of major shells



FIG. 4. Ratio between the theoretical and experimental decay widths calculated as in Fig. 3 as a function of (a) the parameter  $f_2$  and (b) the number of shells  $N_2$ .

## Improved description of the decay width for deformed emitters



FIG. 6. Ratio between the theoretical and experimental decay widths for spherical (circles) and deformed (stars) barriers corresponding to the decays in Table I. The parameters N and f defining the minimal basis are also given.

# Logarithmic derivative condition implies that the ho parameter should be proportional to the Coulomb parameter

PHYSICAL REVIEW C 69, 044318 (2004)

#### Evidence for $\alpha$ clustering in heavy and superheavy nuclei

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We analyze the  $\alpha$  decay between ground states along N-Z chains in deformed heavy and superheavy nuclei, by using the pairing approach. We show that the derivative of the preformation amplitude is practically a constant along any  $\alpha$  chain, while that of the outgoing wave function changes exponentially upon the Coulomb parameter. This leads to the breakdown of the continuity equation and therefore to wrong decay widths. The behavior cannot be explained within the standard shell model. We significantly correct this deficiency by considering an  $\alpha$ -cluster factor in the preformation amplitude, depending exponentially upon the Coulomb parameter. Thus, four-body correlations, connected with the radial shape of the preformation factor, are directly evidenced by the  $\alpha$ -decay systematics. Moreover, this procedure, in principle, fully determines the Q value and is an important development in the  $\alpha$ -decay theory. It also allows us to analyze the relative  $\alpha$ -clustering structure of the emitter. It turns out that the isotopes close to the region N > 126 and superheavy nuclei have a stronger clustering behavior. For superheavy region an additional dependence upon the number of interacting  $\alpha$  particles is necessary.

DOI: 10.1103/PhysRevC.69.044318

PACS number(s): 21.60.Gx, 23.60.+e

## Log. derivative condition is equivalent to the "plateau condition" γ<sub>1</sub>=0

$$\log_{10} \left[ \frac{\Gamma(R)}{\Gamma_{\rm exp}} \right] = \gamma_0 + \gamma_1 R,$$

 $\lg(\Gamma) = lg(P) + lg(2\gamma^2)$ 

 $= a\chi - b\beta \sim const$ 





## By using an ho parameter proportional to the Coulomb parameter we improve both the decay width description $\gamma_0=0$ and "plateau condition" $\gamma_1=0$



$$\beta - \beta_m = (f - f_m)\beta_N = f_1(\chi - \chi_m)\beta_N.$$

FIG. 10. (a) The parameter  $\gamma_0$  vs the neutron number for different even-even  $\alpha$ -chains in Table II. The preformation parameters are  $f_m = 0.83$ ,  $f_1 = 8.0 \ 10^{-4}$ ,  $P_{min} = 0.025$ . (b) The same as in (a), but for the slope parameter  $\gamma_1$ .

## By including beyond mean field correlations on the nuclear surface one obtains in a natural way larger sp tails

PHYSICAL REVIEW C 87, 041302(R) (2013)

#### Shell-model representation to describe $\alpha$ emission

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It is shown that the standard shell-model representation is inadequate to explain cluster decay processes due to a deficient asymptotic behavior of the corresponding single-particle wave functions. A new representation is proposed which is derived from a mean field consisting of the standard Woods-Saxon plus spin-orbit potential of the shell model, with an additional attractive pocket potential of a Gaussian form localized on the nuclear surface. The eigenvectors of this new mean field provide a representation which retains all the benefits of the standard shell model while at the same time reproducing well the experimental absolute  $\alpha$ -decay widths from heavy nuclei.

DOI: 10.1103/PhysRevC.87.041302

PACS number(s): 21.10.Tg, 23.50.+z, 23.60.+e, 23.70.+j

Single particle mean field with alpha-like correlations The potential pocket corresponds to the peak of the microscopic alpha formation amplitude



FIG. 2. Proton and neutron potentials [Eq. (9)] in <sup>216</sup>Rn (solid lines) and the corresponding Woods-Saxon mean-field potentials [19] (dashed lines). The cluster parts are as in Eq. (9) with parameters  $V_{\text{clus}}^n - V_{\text{clus}}^p = V_C$ , b = 1 fm, and  $r_c = 1.3(A^{1/3} + 4^{1/3})$  fm.

### This picture is confirmed by microscopic calculations G. Roepke, et.al., Phys. Rev. C 90, 034304 (2014)



## Pair formation amplitudes are enhanced by surface alpha-like correlations (a)



FIG. 5. Proton (solid lines) and neutron (dashed lines) pairing formation probabilities [Eq. (8)] as functions of the c.m. radius corresponding to the decay process  $^{220}$ Ra  $\rightarrow ^{216}$ Rn +  $\alpha$ . The pocket potential parameters satisfy the conditions (a)  $V_{clus}^n = V_{clus}^p + V_C$  and (b)  $V_{clus}^p = V_{clus}^n = 0$ .

Decay width is strongly enhanced by surface alpha-like correlations and weakly depends on cm radius (a)



FIG. 6. Logarithm of the theoretical decay width divided by the corresponding experimental value as a function of the matching radius for  $^{220}$ Ra  $\rightarrow ^{216}$ Rn +  $\alpha$  (solid line). The first term in Eq. (10) is given by a dashed line, and the second one by a dot-dashed line.

## This approach enabled the description of enhanced E1 transitions from un-natural parity states 2<sup>-</sup>,4<sup>-</sup>,6<sup>-</sup> recently evidenced in <sup>212</sup>Po

Selected for a Viewpoint in Physics PHYSICAL REVIEW LETTERS

week ending 29 JANUARY 2010

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#### Novel Manifestation of $\alpha$ -Clustering Structures: New " $\alpha + {}^{208}$ Pb" States in ${}^{212}$ Po Revealed by Their Enhanced *E1* Decays

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Excited states in <sup>212</sup>Po were populated by  $\alpha$  transfer using the <sup>208</sup>Pb(<sup>18</sup>O, <sup>14</sup>C) reaction, and their deexcitation  $\gamma$  rays were studied with the Euroball array. Several levels were found to decay by a unique *E*1 transition ( $E_{\gamma} < 1$  MeV) populating the yrast state with the same spin value. Their lifetimes were measured by the Doppler-shift attenuation method. The values, found in the range 0.1–1.4 ps, lead to very enhanced transitions,  $B(E1) = 2 \times 10^{-2}$ –1  $\times 10^{-3}$  W.u. These results are discussed in terms of an  $\alpha$ -cluster structure which gives rise to states with non-natural-parity values, provided that the composite system cannot rotate collectively, as expected in the " $\alpha + {}^{208}$ Pb" case. Such states due to the oscillatory motion of the  $\alpha$ -core distance are observed for the first time.

DOI: 10.1103/PhysRevLett.104.042701

PRL 104, 042701 (2010)

PACS numbers: 25.70.Hi, 21.60.Gx, 23.20.-g, 27.80.+w



### Viewpoint: Do alpha particles cluster inside heavy nuclei?

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January 25, 2010 • Physics 3, 8

New excited states have been observed in <sup>212</sup>Po that are associated with a configuration in which an alpha particle is combined with a doubly-magic <sup>208</sup>Pb core.



Figure 1: Partial level scheme for <sup>212</sup>Po. Excited states are labeled by their excitation energy and are given in keV. Parity assignments enclosed in parentheses are consistent with the data, but an opposite parity assignment, while unlikely, cannot be ruled ou... Show more

In the early part of the 20th century as physicists became aware of the existence of the nucleus, they speculated that the nucleus might be composed of  $\alpha$  particles [1]. This picture of the nucleus was eventually supplanted by the shell model, which considers nuclei to consist of neutrons and protons held together by an average



#### PHYSICAL REVIEW C 85, 064306 (2012)

#### Shell model plus cluster description of negative parity states in <sup>212</sup>Po

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The intraband electromagnetic transitions in <sup>210</sup>Po and <sup>210</sup>Pb are well described within the shell model approach. In contrast, similar transitions in <sup>212</sup>Po are one order of magnitude smaller than the experimental values, suggesting the existence of an  $\alpha$ -cluster component in the structure of this nucleus. To probe this assumption we introduced Gaussian-like components in the single-particle orbitals. We thus obtained an enhancement of intraband transitions, as well as a proper description of the absolute  $\alpha$ -decay width in <sup>212</sup>Po. We analyzed the recently measured unnatural parity states  $I^-$  in <sup>212</sup>Po in terms of the collective octupole excitation in <sup>208</sup>Pb coupled to positive parity states in <sup>210</sup>Pb. They are connected by relatively large dipole transitions to yrast positive natural parity states. We described *E1* transitions by using the same  $\alpha$ -cluster component and an effective neutron dipole charge  $e_v = -eZ/A$ . B(E2) values and absolute  $\alpha$ -decay width in <sup>212</sup>Po are simultaneously described within the shell model plus a cluster component depending upon one free strength parameter.

DOI: 10.1103/PhysRevC.85.064306

PACS number(s): 21.60.Jz, 23.20.Js, 23.60.+e, 27.80.+w

# Surface α-clustering in <sup>212</sup>Po explains decay width between ground states

x 10<sup>-3</sup>



Formation probability versus cm radius total: solid line cluster comp: dashes

Log (width / exp. ) versus cm radius

The same cluster amplitude  $\approx$  0.3 explains B(E $\lambda$ ) values and absolute  $\alpha$ -decay width

# Surface $\alpha$ -clustering term with the amplitude $\approx 0.3$ explains large electromagnetic E1 transitions in <sup>212</sup>Po

### B(E2:J+2→J)-values

J'  ightarrow J	$^{210}$ Po $B(E2)_{exp}$	$B(E2)_{th}$	$^{210}$ Pb $B(E2)_{exp}$	$B(E2)_{th}$	$^{212}$ Po $B(E2)_{exp}$	$B(E2)_{th}$
$2 \rightarrow 0$	0.56(12)	6.7	1.4(4)	3.9		9.2
$4 \rightarrow 2$	4.6(2)	12.9	3.2(7)	3.5		20.8
$6 \rightarrow 4$	3.0(1)	8.9	2.2(3)	2.4	13.5(36)	14.4
$8 \rightarrow 6$	1.18(3)	3.9	0.62(5)	1.0	4.60(9)	5.8

### B(E1:I<sup>-</sup>→J<sup>+</sup>)-values

Ι-	$J^+$	$E_{MSM}$ (MeV)	$\frac{E(^{212}\text{Po}(I^-))}{(\text{MeV})}$	$\frac{E_{\exp}(^{212}\text{Po}(I^{-}))}{(\text{MeV})}$	$B(E1)_{th}^{(1)}$ (10 <sup>4</sup> W.u.)	$B(E1)_{th}^{(2)}$ (10 <sup>4</sup> W.u.)	$B(E1)_{exp}$ (10 <sup>4</sup> W.u.)
2-	2+	-0.407	1.236		5	1	
	4+	-0.204	1.907		15	63	
4-	4+	-0.303	1.808	1.744	9	11	25
	6+	-0.107	2.201	1.946	2	4	11
6-	6+	-0.213	1.886	1.787	37	122	66
	8+	-0.490	2.197	2.016	3	8	19
8-	6+	-0.489	1.816	1.751	43	148	200
	8+	-0.215	2.240	1.986	8	24	
10-	8+	-0.360	2.135	2.465	2	1	18

# Surface alpha-like correlations have an universal behavior for alpha decays

PHYSICAL REVIEW C 94, 034319 (2016)

#### Proton-neutron versus α-like correlations above <sup>100</sup>Sn

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It is known that the  $\alpha$  particle reduced width has the largest values in the region above <sup>100</sup>Sn, and this behavior is usually attributed to the proton-neutron correlations. To reproduce the reduced  $\alpha$ -decay width we use an additional pocket-like surface potential in the single-particle mean field that simulates four-body correlations. We show that the strength of this interaction has a universal linear dependence on the experimental reduced width above the double magic nuclei <sup>100</sup>Sn and <sup>208</sup>Pb. Moreover, we demonstrate that proton-neutron pairing correlations have a small influence on this dependence and therefore cannot explain the larger reduced decay widths above <sup>100</sup>Sn. We give an indication of the possibility of detecting Sn +  $n\alpha$  structures as dipole Pigmy-like resonances.

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## Systematics of the surface clustering strength versus the reduced width is given by similar dependencies for Z>50 and Z>82 regions



FIG. 6. The strength of the  $\alpha$ -clustering additional part of the mean-field potential  $V_{\alpha}$  versus the reduced width.

## Anisotropy of the alpha emission in deformed odd-mass nuclei

PHYSICAL REVIEW C

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SEPTEMBER 1992

#### Anisotropy in alpha decay of odd-mass deformed nuclei

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Angular distributions and the corresponding absolute  $\alpha$  decay widths are calculated microscopically in odd axially deformed nuclei. It is found that the angular distributions are mainly determined by the deformation. The available experimental data are well reproduced.

PACS number(s): 23.60.+e, 27.90.+b



# Anisotropy is enhanced by the quadrupole deformation



FIG. 1. Angular distributions corresponding to initial angular momentum projections  $M_i = 1/2$  (solid line), 3/2 (dash-dotted line), and 5/2 (dashed line) for the case  $\beta_2 = 0.2$ ,  $\beta_3 = \beta_4 = 0$ .

# Superheavy nuclei live longer in high spin alpha-decaying 2qp isomeric states with higher hindrance factors

PHYSICAL REVIEW C 76, 044301 (2007)

#### α decay of high-spin isomers in superheavy nuclei

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Hindrance factors corresponding to  $\alpha$  decay from two quasiparticle isomeric high K states are evaluated in superheavy nuclei. We found that the hindrance factors are very sensitive to the deformations and, therefore, they may constitute a powerful tool to extract spectroscopic information in these nuclei. The hindrance factors turn out to be very large, specially for nonaligned configurations. This indicates that if one of such states is reached the parent nucleus may become isomeric. It is also possible that  $\alpha$  decay may not proceed through ground state to ground state chains but rather through excited states.

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PACS number(s): 21.10.Jx, 21.60.Gx, 23.60.+e, 27.90.+b

# Hindrance factors (ratio between reduced widths of ground and excited states)

TABLE III. Hindrance factors for  $10^+_{\pi}$  and  $12^+_{\nu}$  high-spin 2qp – isomers together with ground state spectroscopic factors for  $\alpha$ emitters with Z > 110.

Nucleus	$\beta_2$	$HF[10_{\pi}^{+} =$	$HF[12^+_{\nu} =$	$S_0$
		$\frac{2}{2}(9/2) \otimes \frac{11}{2}(11/2)$	$\frac{11}{2}$ (11/2) $\otimes$ $\frac{13}{2}$ (13/2)]	
280112	-0.10	3.43 10 <sup>1</sup>	2.47 10 <sup>1</sup>	$5.55 \ 10^{-3}$
<sup>282</sup> 112	-0.10	3.99 10 <sup>1</sup>	$2.47 \ 10^{1}$	$5.47 \ 10^{-3}$
<sup>284</sup> 112	-0.10	$4.92 \ 10^{1}$	$2.48 \ 10^{1}$	$5.32  10^{-3}$
286112	-0.10	6.18 10 <sup>1</sup>	2.30 10 <sup>1</sup>	$4.81 \ 10^{-3}$
<sup>288</sup> 112	-0.10	8.35 101	$2.15\ 10^{1}$	4.31 10-3
<sup>282</sup> 114	-0.10	3.92 101	2.57 10 <sup>1</sup>	6.05 10-3
<sup>284</sup> 114	-0.10	4.67 10 <sup>1</sup>	$2.51 \ 10^{1}$	$6.00 \ 10^{-3}$
<sup>286</sup> 114	-0.10	5.53 10 <sup>1</sup>	$2.40\ 10^{1}$	$5.62  10^{-3}$
<sup>288</sup> 114	-0.10	7.16 10 <sup>1</sup>	2.35 10 <sup>1</sup>	$5.25 \ 10^{-3}$
<sup>290</sup> 114	-0.10	9.66 10 <sup>1</sup>	$2.21 \ 10^{1}$	$4.67 \ 10^{-3}$
<sup>284</sup> 116	-0.10	$4.86\ 10^{1}$	$2.71 \ 10^{1}$	$6.75 \ 10^{-3}$
<sup>286</sup> 116	-0.10	5.72 10 <sup>1</sup>	$2.82\ 10^{1}$	$6.71 \ 10^{-3}$
287116	-0.10	6.80 10 <sup>1</sup>	2.66 10 <sup>1</sup>	$6.40\ 10^{-3}$
<sup>290</sup> 116	-0.10	7.50 10 <sup>1</sup>	2.58 10 <sup>1</sup>	$5.40 \ 10^{-3}$
<sup>292</sup> 116	-0.10	$1.15 \ 10^2$	$2.33 \ 10^{1}$	$5.11 \ 10^{-3}$
286118	-0.10	5.80 10 <sup>1</sup>	2.91 10 <sup>1</sup>	7.30 10-3
<sup>287</sup> 118	-0.10	6.71 10 <sup>1</sup>	3.15 10 <sup>1</sup>	$7.26  10^{-3}$
<sup>290</sup> 118	-0.10	$8.14 \ 10^{1}$	3.13 10 <sup>1</sup>	$6.97 \ 10^{-3}$
<sup>292</sup> 118	-0.10	$1.02\ 10^2$	$2.93 \ 10^{1}$	$6.30  10^{-3}$
<sup>294</sup> 118	-0.10	1.39 10 <sup>2</sup>	$2.70\ 10^{1}$	$5.59\ 10^{-3}$
<sup>287</sup> 120	-0.10	6.36 10 <sup>1</sup>	3.00 10 <sup>1</sup>	$6.82 \ 10^{-3}$
<sup>290</sup> 120	-0.10	7.82 10 <sup>1</sup>	3.32 10 <sup>1</sup>	7.22 10-3
<sup>292</sup> 120	-0.10	9.34 10 <sup>1</sup>	3.34 10 <sup>1</sup>	$6.82 \ 10^{-3}$
<sup>294</sup> 120	-0.10	$1.21 \ 10^2$	3.33 10 <sup>1</sup>	$6.33 \ 10^{-3}$
<sup>296</sup> 120	-0.10	1.66 10 <sup>2</sup>	3.16 10 <sup>1</sup>	$5.63 \ 10^{-3}$

## Probing shape coexistence by alpha decay

PHYSICAL REVIEW C 90, 061303(R) (2014)

#### Probing shape coexistence by $\alpha$ decays to 0<sup>+</sup> states

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We analyze the  $\alpha$ -decay fine structure to excited  $0_2^+$  states in Hg and Rn isotopes. These states are described as minima in the potential energy surface (PES) provided by the standard deformed Woods-Saxon plus pairing approach. We also investigate  $\alpha$  decay *from* the excited state  $P(0_2^+)$  in the parent nucleus by evaluating the corresponding hindrance factor (HF). By analyzing the experimental HF's we find the remarkable property that the ground and excited states  $D(0_1^+)$  and  $D(0_2^+)$  in the daughter nuclei are occupied with almost equal probabilities if there is no excited  $P(0^+)$  states in the parent nucleus. Moreover, if there exists an excited state  $P(0_2^+)$  then the occupation probability of this state is 25%.

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PACS number(s): 21.10.Tg, 21.60.Gx, 23.60.+e, 27.70.+q

# Minima on the potential energy surface (PES) have different quadrupole deformations



FIG. 1. (Color online) PES of <sup>182</sup>Hg. Local minima for the ground state (gs) ( $\beta_2 = -0.13$ ) and excited  $0^+_2$  state ( $\beta_2 = 0.27$ ) are shown by circles.

# Spectroscopic factor versus daughter quadrupole deformation



## The key ingredient is the overlap between BCS wave functions

$$|\psi_k^P|\psi_{k'}^D| = \sum_{m>0} \left[ u_m^P(\beta_2^{(k)}) u_m^D(\beta_2^{(k')}) + v_m^P(\beta_2^{(k)}) v_m^D(\beta_2^{(k')}) \right].$$

FIG. 2. Spectroscopic factor for the transition <sup>180</sup>Hg(gs)  $\rightarrow$ <sup>176</sup>Pt(0<sup>+</sup><sub>2</sub>) +  $\alpha$  between BCS states with k = 1 in Eq. (4). The parent (initial) state corresponds to an oblate deformation with  $\beta_2^{(1)}B = -0.12$ . The daughter (final) state carries a deformation  $\beta_2^{(2)}$  as given in the abscissa. The dashed curve was obtained by considering the overlap between the parent and daughter BCS states as unity (i.e.,  $\langle P | D \rangle = 1$ ) while for the full curve that overlap is the one provided by our calculation. Vertical lines denote the deformations provided by the PES calculation for the minima corresponding to the  $0^+_1$  (k' = 1) and  $0^+_2$  (k' = 2) states.

## **Shape mixing of wave functions**

$$\begin{aligned} \left|\varphi_{1}^{A}\right\rangle &= X_{A}\left|\psi_{1}^{A}\right\rangle + Y_{A}\left|\psi_{2}^{A}\right\rangle, \\ \left|\varphi_{2}^{A}\right\rangle &= -Y_{A}\left|\psi_{1}^{A}\right\rangle + X_{A}\left|\psi_{2}^{A}\right\rangle, \ A = P, D, \end{aligned}$$

## in terms of the mixing angle

$$\begin{cases} X_A^2 \\ Y_A^2 \end{cases} = \frac{1}{2} \pm \delta_A.$$

# Hindrance factor in terms of spectroscopic factors

**experimental** 
$$H_{exp}(k) = \left| \frac{\left\langle \varphi_1^D | \hat{T} | \varphi_k^P \right\rangle}{\left\langle \varphi_2^D | \hat{T} | \varphi_k^P \right\rangle} \right|^2$$

$$\left\langle \psi_{k'}^{D} \left| \hat{T} \right| \psi_{k}^{P} \right\rangle \equiv T_{k'k} = \sqrt{\langle S_{kk'} \rangle}.$$

÷.

**theoretical** 
$$H_{th}(k) = \left| \frac{\left\langle \psi_1^D \left| \hat{T} \left| \psi_k^P \right\rangle \right|^2}{\left\langle \psi_2^D \left| \hat{T} \left| \psi_k^P \right\rangle \right|^2} \right|^2 = \left| \frac{T_{1k}}{T_{2k}} \right|^2$$

## **Spectroscopic factor**

$$S_{kk'} = \int_0^{R_{w}} R^2 dR \int d\hat{R} |\mathcal{F}_{kk'}(\mathbf{R})|^2$$
  
$$\approx \sum_{N_{w}L_{w}} W_{N_{w}L_{w}}^2 (\beta_2^{(k)}, \beta_2^{(k')}).$$

# is the integral of the formation amplitude squared, written in terms of spherical orbitals

$$\mathcal{F}_{kk'}(\mathbf{R}) = \sum_{L_{\alpha}N_{\alpha}} W_{N_{\alpha}L_{\alpha}} \left(\beta_2^{(k)}, \beta_2^{(k')}\right) \Phi_{N_{\alpha}L_{\alpha}}^{(4\lambda)}(\mathbf{R}),$$

# Experimental HF determines shape mixing angles for parent and daughter nuclei

TABLE I. Quadrupole deformation of the  $0_1^+$  (third column) and  $0_2^+$  state (fourth column). Experimental HF (fifth column) and theoretical HF for  $0_1^+$  (sixth column) and  $0_2^+$  states (seventh column). In the last column is given the mixing parameter (10). The first line for each case corresponds to parent and the second line to daughter nucleus.

No.	Parent Daughter	$\begin{array}{c} \beta_2^{(1)}(0_1^+) \\ \beta_2^{(2)}(0_1^+) \end{array}$	$\begin{array}{c} \beta_2^{(1)}(0_2^+) \\ \beta_2^{(2)}(0_2^+) \end{array}$	H <sub>exp</sub> (1)	$H_{th}(1)$	$\mathbf{H}_{th}(2)$	$\delta_P \\ \delta_D$
1	<sup>180</sup> Hg <sup>176</sup> Pt	$-0.12 \\ 0.17$	0.24	16.1	64.0		0.5 0.03
2	<sup>182</sup> Hg <sup>178</sup> Pt	- 0.13 0.25	0.27 0.18	4.8	0.1	1.1	0.25 0
3	<sup>184</sup> Hg <sup>180</sup> Pt	- 0.13 0.26	0.25 0.18	1.9	0.06	1.7	0.25 0
4	<sup>202</sup> Rn <sup>198</sup> Po	0.09 0.07	- 0.15	25.2	810.9		0.5 0.007

## 3. Heavy cluster emission

J. Phys. G: Nucl. Part. Phys. 20 (1994) 1483-1498. Printed in the UK.

### Microscopic description of cluster decay

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Abstract. A microscopic description of the heavy cluster spontaneous emission problem is presented. A realistic large single-particle basis and pairing two-body interaction are used in computing the cluster preformation amplitude. The barrier penetration process is treated within the wKB approximation. Applications are presented for the <sup>14</sup>C decay from Ra isotopes and <sup>12</sup>C decay from <sup>114</sup>Ba. The calculated total widths are about one order of magnitude smaller than the currently accepted experimental values.

## **Cluster formation amplitude**



Figure 4. The preformation amplitude given by relation (2.34) as a function of the CM rad between daughter nucleus and cluster  ${}^{114}\text{Ba} \rightarrow {}^{12}\text{C} + {}^{102}\text{Sn}$ .

## Cluster spectroscopic factor and decay width

Table 1. Spectroscopic factors  $S_{th}$ , according with (3.1) and total widths for the emission of <sup>14</sup>C. For <sup>114</sup>Ba  $\rightarrow$  <sup>12</sup>C + <sup>102</sup>Sn decay the Q-value has been fixed at Q = 20.79 MeV [17]. The meaning of the factors (-xy) in parenthesis is the following:  $\times 10^{-xy}$ .

Decay		Sth.	$\Gamma_{exp}$ (MeV)	Γ <sub>th</sub> (MeV)
<sup>222</sup> Ra →	<sup>14</sup> C + <sup>208</sup> Pb	2.3(-10)	4.1(-33)	3.5(-34)
$^{224}$ Ra $\rightarrow$	<sup>14</sup> C + <sup>210</sup> Pb	1.3(9)	6.3(-38)	3.4(-39)
<sup>226</sup> Ra →	$^{14}C + ^{212}Pb$	1.8(-9)	2.9(-43)	5.8(-43)
<sup>114</sup> Ba →	$^{12}C + ^{102}Sn$	1.8(-7)		4.4(-27)

PHYSICS REPORTS



Physics Reports 294 (1998) 265-362

### Microscopic theory of cluster radioactivity

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# Conclusions

Our collaboration reported important results for

- proton emission systematics

### PRL & Phys. Rep. with ~ 150 citations

- microscopic description of the alpha emission

### 4 Phys. Rev. C with ~ 200 citations

- microscopic description of heavy cluster decays

### Phys. Rep. with ~ 300 citations

- anisotropic alpha emission from odd-mass nuclei

### 2 Phys. Rev. C with ~ 100 citations

- enhanced formation of superheavy nuclei in isomeric states

### Phys. Rev. C with ~ 60 citations

- shape coexistence evidenced by alpha decay

### Phys. Rev. C with ~ 15 citations

# Happy birthday Roberto !

