

The Mössbauer Effect in ^{57}Fe

Laboratory Exercise in Nuclear Physics

Before starting the lab exercise you should read chapter 10.9 in K.S. Krane *Introductory Nuclear Physics*.

1 Introduction

When an excited nucleus decays by emitting a gamma ray, this gamma ray might be absorbed by another nucleus, but only if there in that nucleus exists a transition with exactly the correct energy. This resonant absorption can usually not take place if the nuclei are of the same kind, since the nuclei will recoil. The recoil will cause the energy of an emitted gamma ray to be slightly lower than the energy of the corresponding transition, and if a gamma ray is to be absorbed, its energy has to be slightly higher than the transition energy. This shift in energy is several orders of magnitude larger than the intrinsic width of the resonance, and therefore the resonant absorption of a emitted gamma ray usually is not possible, even if the absorbing and emitting nuclei are of the same kind. The shift caused by the recoil of nuclei is also known from atomic physics, but there the shift is much smaller since the transition energies and therefore the recoil energies are much lower than in the case of nuclear physics.

In 1958, Rudolph Mössbauer discovered that some of the nuclei in the crystal that he studied could emit and absorb a light quantum without absorption or emission of a phonon. This means that the crystal will be in the same internal state before and after the event. The recoil is taken up by the crystal as a whole, and not by the individual atom, which makes the recoil energy, and thereby the shift in energy for the gamma ray, immeasurably small. Therefore, a photon emitted in such a way can be absorbed by an identical nucleus, provided that also this nucleus is bound in a crystal.

The probability for this absorption without recoil increases with decreasing temperature, and usually only a small fraction of the gamma rays are absorbed and emitted recoilless at room temperature. However, in the case of ^{57}Fe in steel, which will be studied in this lab exercise, as much as 14% of the absorbed gamma photons are absorbed recoil free at room temperature.

2 Background

Assume that a nucleus absorbs a gamma quantum. The conservation of energy and momentum gives:

$$E_f + E_R = E_i + E_\gamma \quad (1)$$

$$p_R = p_\gamma \quad (2)$$

If these two equations are combined, the result is:

$$\Delta E = E_f - E_i = E_\gamma - \frac{E_\gamma^2}{2Mc^2} \quad (3)$$

This means that the energy has been shifted an amount ΔE towards higher energies. However, because of the natural line width, gamma ray absorption may occur for energies that differ slightly from this value. Every excited state has a life time, τ , and therefore a natural line width $\Gamma = \hbar/\tau$. For low-lying states, this width is between 10^{-6} and 10^{-3} eV.

In addition to the natural line width, there is also the Doppler broadening. The Doppler width is caused by the thermal motion of the absorbing and emitting nuclei, and can be expressed as:

$$\Delta = 2\sqrt{\ln 2} E_\gamma \sqrt{\frac{2kT}{Mc^2}} \quad (4)$$

The Doppler width is usually a couple of orders of magnitude larger than the natural line width. The two widths combine to give a total line width Γ^* , where $\Gamma^{*2} = \Gamma^2 + \Delta^2$. The gamma ray absorption cross section is then:

$$\sigma(E_\gamma) = \sigma_0 \frac{(\Gamma^*/2)^2}{[E_\gamma - (\Delta E + E_R)]^2 + (\Gamma^*/2)^2} \quad (5)$$

where σ_0 is the absorption cross section for gamma rays of energy $\Delta E = E_R$.

In a simplified picture, the atoms in the crystal can be assumed to move with velocities that are Maxwell distributed:

$$f(v_x) = e^{-\frac{mv_x^2}{2kT}} \quad (6)$$

This movement results in a Doppler shift in any emitted gamma ray:

$$E'_\gamma = E_\gamma \left(1 + \frac{v}{c}\right) \quad (7)$$

which gives an energy distribution that is given by:

$$f(E) = e^{-\frac{mc^2}{2kT} \left(1 - \frac{E'_\gamma}{E_\gamma}\right)^2} \quad (8)$$

For the absorption, the distribution is centered around $E_\gamma + \Delta E$, and for the emission around $E_\gamma - \Delta E$. The area of the overlapping part of the peaks (see figure 1) can be changed by changing the temperature of the source and/or the absorber.

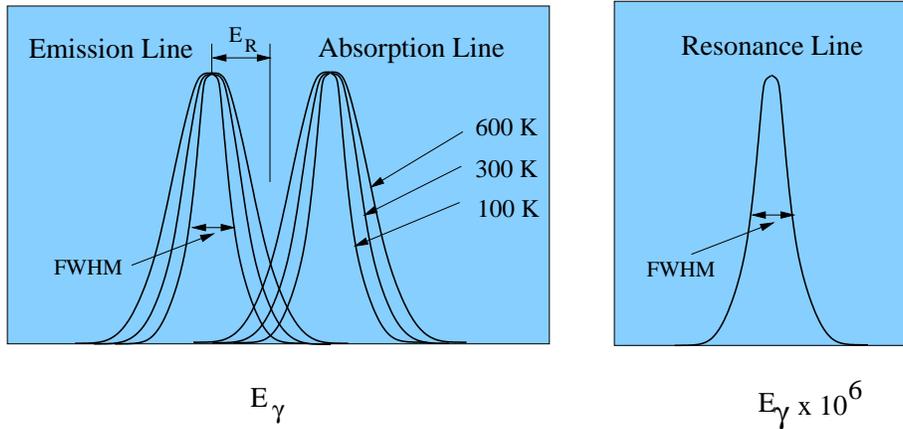


Figure 1: The left picture shows how the width of the emission and absorption lines will change with increasing temperature. E_R represents the recoil energy. A resonance line, which is not Doppler broadened, would only be a straight line in this plot. If the energy scale is a million times enlarged, a resonance line will look as in the right plot.

2.1 The Mössbauer Effect

When Rudolf Mössbauer studied the resonant absorption of ^{191}Ir , he found something interesting. He looked at the absorption as a function of temperature. When decreasing the temperature of the source and absorber he expected to see a decreased absorption (at lower temperature the Doppler broadened peaks of figure 1 overlap less). But to his surprise he found an *increase* of the absorption. He could explain the new effect when he realized that part of the nuclei can emit or receive a gamma photon without recoil.

From the above we see that the Mössbauer effect has two characteristic features:

- The absorption is recoil free.
- The emission and absorption lines have no Doppler broadening.

3 The Nuclear Multipole Moment

In order to simplify the calculations that will be done during the lab exercise, a short description of the nuclear moments and their interaction with the electromagnetic field is included.

Both the magnetic vector potential and the electric potential are given by integrals over current and charge distributions:

$$\bar{A}(\bar{r}) = \frac{\mu_0}{4\pi} \int \frac{\bar{j}(\bar{r}') dv'}{|\bar{r} - \bar{r}'|} \quad (9)$$

$$V(\bar{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\bar{r}') dv'}{|\bar{r} - \bar{r}'|} \quad (10)$$

$$(11)$$

By Taylor expanding the denominator, a so-called multiple expansion of the potential is obtained.

$$\frac{1}{|\bar{r} - \bar{r}'|} = \frac{1}{r} + \frac{1}{r^3} \bar{r} \cdot \bar{r}' + \dots \quad (12)$$

This can be used in combination with Gauss theorem in the quantum mechanical case to show that:

$$\bar{A}(\bar{r}) = \frac{\mu_0}{4\pi} \frac{\bar{\mu} \times \bar{r}}{r^3} \quad (13)$$

$$V(\bar{r}) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r} \int \rho(\bar{r}') dv' + \frac{1}{r^2} \int \rho(\bar{r}') r' \cos \theta dv' + \frac{1}{2r^3} \int \rho(\bar{r}') r'^2 (3 \cos^2 \theta - 1) dv' + \dots \right] \quad (14)$$

where

$$\bar{\mu}(\bar{r}') = \frac{e}{2m} \int \psi^*(\bar{r}') l \psi(\bar{r}') dv' \quad (15)$$

$$\rho(\bar{r}') = e \psi^*(\bar{r}') \psi(\bar{r}') \quad (16)$$

The dominating term in the expansion of the vector potential will be $\bar{\mu}$. If the wave function is an eigenstate of l_z , only this term will survive the integration. Furthermore, we know that it's eigenvalue is m_l . Therefore:

$$\mu_z = \frac{e\hbar}{2m} m_l \quad (17)$$

For $m_l = l$ we define:

$$\mu = \frac{e\hbar}{2m} l \quad (18)$$

where m is the nuclear mass. The *nuclear magneton* is defined as:

$$\mu_N = \frac{e\hbar}{2m_p} \quad (19)$$

where M_p is the mass of a proton. In a similar way as for the spin of the electron, a g-factor is introduced so that:

$$\mu = g_l \mu_n l \quad (20)$$

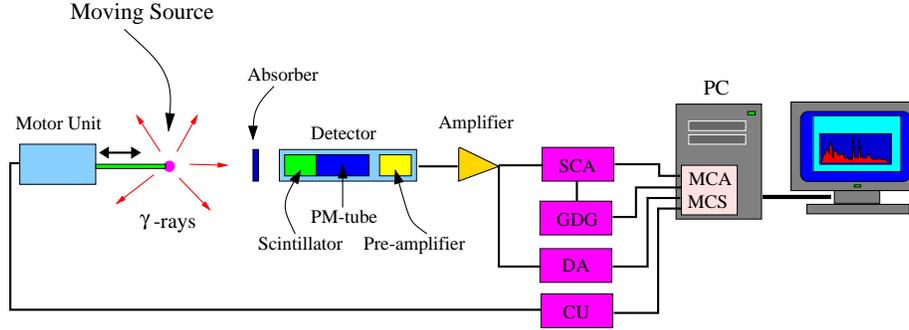


Figure 2: The equipment used to study the Mössbauer effect.

For the total angular momentum of the nucleus, I , the expression is analogically:

$$\mu = g_I \mu_n I \quad (21)$$

The energy for a magnetic dipole in a magnetic field is given by:

$$E_M = -\vec{m} \cdot \vec{B} \quad (22)$$

This gives in our case:

$$E_M = -g_I \mu_N B \bar{I} \cdot \bar{e}_z = -\frac{\mu m_I B}{I} \quad (23)$$

For the electrical potential, the first term gives the contribution from the total charge. The second term is zero, since the integrand is an odd function. The third term gives the quadrupole moment of the nucleus (see Krane). The electrical quadrupole moment gives a splitting of the energy levels if the nucleus is placed in an electrical field gradient. This gradient may for example stem from the atoms in the lattice, and is then dependent on the symmetry of the lattice. If the charge distribution is in an external potential, V_e , the electrostatic energy is given by:

$$W = \int \rho(\vec{r}) V_e(\vec{r}) dv \quad (24)$$

If the external potential is expanded in a Taylor series, the result is:

$$E_Q = \frac{1}{4} eQ \frac{3m_I^2 - I(I+1)}{I(2I-1)} \left(\frac{d^2 V_e}{dz^2} \right)_{z=0} \quad (25)$$

4 The Lab Exercise

The experimental equipment includes a source and an absorber. The absorber is placed in between the detector and the source. The speed of the source is varied in a well-controlled manner, so that the energy of the emitted gamma ray

also changes, because of the Doppler effect. The variation will be over a narrow interval, which is some ten natural line widths. When a resonance occurs in the absorber, the number of detected gamma rays will decrease. By gating on this energy, the SCA gives a digital pulse every time a light quantum with the correct energy hits the detector. Therefore it is enough to count how the number of pulses per unit of time decreases at the resonance. In a MCS spectrum, this can be seen as a decrease in intensity in the channel that corresponds to this time. Since the time dependence of the velocity is known, the velocity that corresponds to this channel is also known, and therefore the Doppler shift of the corresponding radiation can be calculated. For this to work in practice, the velocities must have the same duration. Otherwise, one would get a varying intensity. Another way of saying this is:

$$\frac{dv}{dt} = \text{konst}$$

Therefore, the position of the source must be a quadratic function of the time. In this lab exercise, the velocity has a triangular shape as a function of time, which means that the source will have the same velocity twice during one period of motion. Therefore, the spectrum will be doubled.

4.1 Equipment

4.1.1 Single Channel Analyzer (SCA)

The single channel analyzer determines whether the height of its input signal is between two limits. If the signal is lower than the lower limit or higher than the upper limit, no output signal is given. Otherwise, a digital pulse is given as output. The interval inside which the signal is accepted is called the *gate* of the analyzer. The levels are adjusted using the potentiometers called LLD (Lower Level Discriminator) and ULD (Upper Level Discriminator), respectively. The SCA can also be set to another mode, in which the size of the gate is set to a fixed value. The entire gate can then be moved. In either case, the output signal isn't activated until the input signal has passed the lower trigger level the second time. This means that the output signal will be delayed with respect to the input signal.

4.1.2 Gate and Delay Generator (GDG)

This unit is triggered by an incoming pulse, and creates a gate pulse with variable length as output. This can be used to create a digital pulse of variable length as output. As the name of the device suggests, it can also be used to delay the pulse.

The output signal can be used in other units, which demand digital input signals with a certain pulse length as input. In this lab exercise, the signal will be used to create a coincidence, i.e. make a multi channel analyzer (see below) only accept energy pulses that fall within an interval which is determined by a single channel analyzer.

4.1.3 Delay Amplifier (DA)

To create a delayed pulse, cables of a well-defined length are often used. They can also be integrated in certain amplifiers, which in that case will provide amplification as well as the possibility to choose between some different delay times.

4.1.4 Amplifier

The amplifier is designed to increase the amplitude of the input signal, without losing the information contained in the signal. The amplifier that will be used in the lab exercise has two outputs, one in the front and one at the back of the box. This makes it possible to divide the signal in two branches, which can be treated differently.

4.1.5 Multi-Channel Analyzer (MCA)

In this lab exercise, we use a MCA card which is mounted inside a PC. On the back of this device, there are two inputs that are of interest. The energy signal should be connected to the input marked ADC. If a digital pulse is connected to the GATE input, only those energy signals that are coincident with the GATE signal will be registered by the MCA. The **Maestro** software will be used as an interface for the acquisition system.

4.1.6 Multi-Channel Scaler (MCS)

A unit that only counts the number of pulses on the input is called a *scaler*. Usually, the amplitude of the input signal must be within a certain interval to be registered. A multi channel scaler can be regarded as a multi channel analyzer, in which every channel is an independent counter. If every channel is active during a given interval of time, after which the closest higher channel is activated instead, it is possible to display the number of input pulses as a function of the time. To use the MCS on the PC, a special program is used. The input signal to the MCS-function is connected to the TTL-input.

4.1.7 Velocity Control (CU)

The DAC output on the PC is connected to a driving unit, so that the computer can control the movement of the source. In this case, the CU will send a triangular signal to the engine. From the engine, a signal is fed back into the driving unit. This signal is the difference between the signal emitted by the computer and the true movements of the source. By checking this signal on an oscilloscope, it is possible to deduce whether the source is moving the way it should.

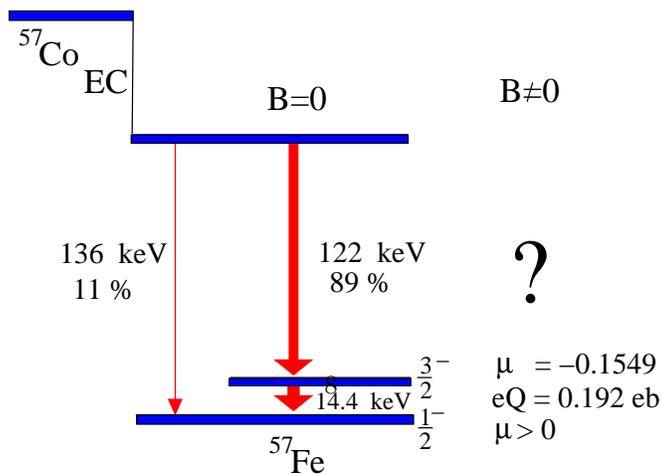
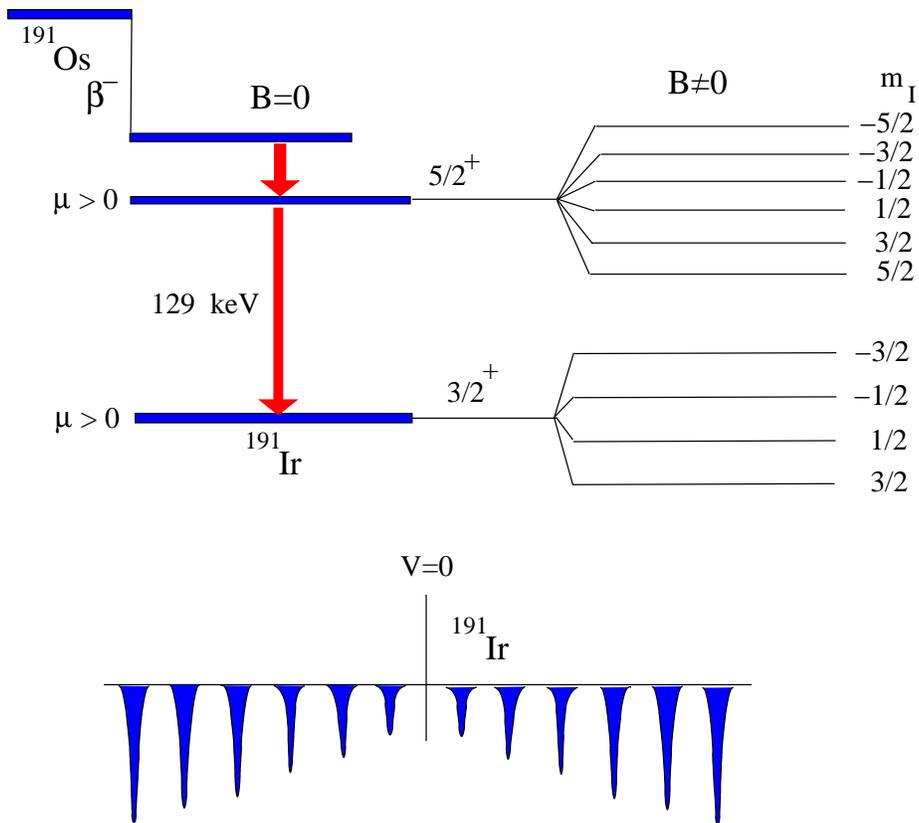


Figure 3: The figure shows the level scheme for ^{57}Fe .

4.2 Assignments

The task of this lab exercise is to measure the self-absorption of the 14.4 keV line in ^{57}Fe . As can be seen in figure 3, this transition is from a $\frac{3}{2}^-$ state that is populated after electron capture (EC) in ^{57}Co . The final state of the transition is the ground state, which has the spin and parity $\frac{1}{2}^-$. Resonance Firstly, set the gate on the single-channel analyzer. The MCA program in the PC can be used for doing this. When the gate is set, connect the signal from the SCA to the PC, and start the multi scaler program. The measurements will be done with three different absorbers:

- Stainless steel
- Natural iron
- FeSO_4

These three samples contain ^{57}Fe in different environments. The first one, stainless steel, does not give any splitting of the energy levels. The natural iron is ferro magnetic, and the magnetic field inside a domain is very strong. The magnetic moment of the nucleus will give a splitting of the energy levels in this field. The spin and parity of the initial and final states are known, but not the magnitude of μ in the ground state, which gives two possibilities when it comes to identifying the transitions.

In the last specimen, FeSO_4 , the nucleus sits in an environment with a electric field gradient. The electrical quadrupole moment will because of this split the energy levels in a different way than for the magnetic dipole moment. This is known:

- $\mu = -0.1549\mu_N$ in the 14.4 keV excited state.
- $\mu > 0$ in the ground state
- $eQ = 0.192 \text{ eb}$ for the excited state.

Solve the following problems:

- Calculate the life time for the 14 keV state by measuring with the first absorber.
- Change to the second absorber. Which transitions are now possible and why?
- Calculate the magnetic field and μ for the ground state.
- Finally, how are the levels split in the case of the last absorber?
- Calculate the electric field gradient.