Solutions to the Written Exam, Radiation Protection, Dosimetry, and Detectors (SH2603), Oct 23, 2008, KTH, Stockholm

Section A

1. The fission products will be distributed in the neutron rich part (i.e. to the right of the stability line) of the nuclide chart from around A=70 to around A=160. The fission products will end up here because the stability line is not straight. Uranium-235 has more neutrons (143) than protons (92), and the neutron/proton ratio of the fission products will be distributed around approximately this neutron-proton ratio. Between A=70 and A=160 the stable nuclei have a lower neutron/proton ratio than 143/92, and therefore the fission products will typically be more neutron rich than stable nuclides of the same mass.

   Because of the neutron excess, the fission products will typically decay by β^-, step-by-step, until they reach a stable nuclide. Sometimes, this beta decay is associated with gamma emission (from internal transitions in nuclei), and sometimes with (beta-delayed) neutron emission.

2. Yes, the reaction is possible. From the mass table, we get the masses for hydrogen and deuterium;
   m(^{1}H)=1.0078250321[u]
   m(^{2}H)=2.0141017780[u].
   The neutron mass is 1.008664915[u].

   The Q-value is now:

   $$Q = \Delta m \cdot c^2 = (1.0078250321 + 1.008664915 - 2.0141017780)[u] \cdot c^2 = \quad (1)$$
   $$0.0023882[u] \cdot c^2 = 0.0023882 \cdot 931.5[MeV] = 2.22[MeV], \quad (2)$$

   where we have used that: 1[u]=931.5 MeV/c^2. In this reaction, the excess energy (2.22 MeV) is emitted in the form of a gamma photon.

3. The half-life $t_{1/2}$ for Po-210 is 138.38 days. We know that the activity A decreases exponentially over the time t:

   $$A = A_{t=0} e^{-\lambda t} \quad (3)$$

   Solving for t:

   $$t = -ln(\frac{A}{A_{t=0}}) \cdot \frac{1}{\lambda} = -ln(\frac{A}{A_{t=0}}) \cdot \frac{t_{1/2}}{ln(2)} \quad (4)$$

   With $A/A_0 = 40/63.42$, we get $t = 92.014$. This means that the source should be delivered 92 days after the 3:rd of March, i.e. on the 3:rd of June 2008.

4. From the *Table of Isotopes* we can identify (among others) the following six energies as internal transitions of Cs-133 (Ba-133 decays by electron capture, and forms Cs-133): 356.0 keV, 276.4 keV, 53.1 keV, 383.9 keV, 302.9 keV, and 223.2 keV.

5. We remember that nuclear matter has constant density, so that we can write, for the radius:
$$R = R_0 \cdot A^{1/3} \tag{5}$$
The cross section, S, is then:
$$S = \pi R^2 = \pi R_0^2 A^{2/3} \tag{6}$$
If we cannot remember the value for R_0, we can use that the cross section for Ca-40 is $0.53 \cdot 10^{-28} m^2$:
$$R_0 = \sqrt{\frac{S}{\pi A^{2/3}}} = \sqrt{\frac{0.53 \cdot 10^{-28}}{\pi 40^{2/3}}} = 1.2 \cdot 10^{-15} [m]. \tag{7}$$
For U-235, we get:
$$S = \pi (1.2 \cdot 10^{-15})^2 235^{2/3} = 1.72 \cdot 10^{-28} = 1.72 [barn] \tag{8}$$
The result, 1.72 [barn], is surprisingly close to the measured cross section values for the most common reactions at high neutron (e.g. 2 MeV) energy. At low energy however, this simple approximation give very poor results.

6. The fission ionisation chamber operates by combining a fissile material with a standard ionisation (gas-filled) chamber. By appying a thin layer of a fissile material (thermal neutrons will induce fission in e.g. U-235 and Pu-239) on the inner side of the wall of the detector container, an incoming neutron will penetrate the outer shell of the detector, but induce fission in the layer of fissile material. Since typically one of the (highly charged) outgoing fission products will be emitted from the thin layer into the gas of the ionisation chamber it will ionise the gas. Then, free electrons will be accelerated towards the high voltage (positive) electrode, and a pulse will be generated, as a response to the incoming neutron.

7. The low energy photons (around 80 keV) are characteristic X-rays, emitted from lead. When the high energy gamma photons (1173 keV and 1332 keV from the Co-60 source) interacts with the lead, electrons from the inner atomic shell(s) can be removed from its state by photo-effect interaction or Compton scattering by the incoming gamma photons. The hole left in the atomic shell will be filled by another electron, and a photon (X-ray) will be emitted at the same time. If the Co-60 source is not present, the electrons will not be removed from the inner atomic shell(s), and no X-ray photons will be emitted.

8. By looking at the Bethe-Bloch formula, we see that the stopping power (dE/dx) only depends on the material, the velocity of the projectile, and the charge of the projectile. For the same velocity, and the same material, the stopping power will only depend on the charge, z, like this:
$$-\frac{dE}{dx} = z^2 \cdot C, \tag{9}$$
where C is a constant. The charge ratio between the fission product Kr-92 and the alpha particle is 36/2. If the stopping power is 0.8 MeV/cm for an alpha particle, we get, for Kr-92, a stopping power of $0.8 \cdot (36/2)^2 = 259.2$

MeV/cm. It should be pointed out here that it is unrealistic that a fission product has as high velocity as an alpha particle of over 5 MeV. The condition of same velocities was set to make the problem simpler.

9. The nuclide Cf-252 decays by alpha emission (97% probability) and spontaneous fission (3%). The particles emitted directly from the Cf-252 source are therefore **alpha particles**, **fission fragments**, and **neutrons** (since spontaneous fission is always associated with neutron emission). (The various decay products will in turn emit various particles, including gamma photons, beta electrons, neutrinos, and delayed neutrons).

10. We have (according to the problem text), for the density of air, ρ, as a function of altitude above sea level, h:

$$\rho(h) = \rho_0 e^{-z \cdot h}, \tag{10}$$

where ρ_0 is the air density at sea level, and z is a constant. From the problem we get:

$$0.5 = e^{-z \cdot 5000} \tag{11}$$

$$z = \frac{ln(2)}{5000}[m^{-1}]. \tag{12}$$

By integrating $\rho(h)$ from $h = 0$ to infinity, ge get:

$$\int_0^\infty \rho(h)dh = \rho_0 \int_0^\infty e^{-z \cdot h} dh = \tag{13}$$

$$\frac{\rho_0}{-z}(e^{-\infty} - e^0) = \frac{\rho_0}{z} \tag{14}$$

This means that we can (for this problem) replace the exponentially decreasing density atmosphere with a layer of constant density (ρ_0), and with a thickness of $1/z$, e.g. $5000/ln(2) = 7213.5$ [m].

Using the absorption coefficient at 1.25 MeV (a reasonable approximation of the gamma energies emitted from the Co-60 source), we get:

$$\frac{I}{I_0} = e^{-\frac{\mu}{\rho}\rho x} = e^{-5.687 \cdot 10^{-2} \cdot 1.205 \cdot 10^{-3} \cdot 721350} = 3.4 \cdot 10^{-22}. \tag{15}$$

Only 1 in $3.4 \cdot 10^{-22}$ photons will be able to pass through the atmosphere. It is clear that the atmosphere effectively stops the gamma photons. Even with an extremely strong source, say 1000 Ci, only about 10^{13} gamma photons are emitted from the source per second, so we would have to wait for hundreds of years (in the order of 10^9 seconds, not considering the solid angle) before any gamma was able to pass through the atmosphere and reach outer space.

Section B

1. The radioisotope ^{11}C is a β^+-emitter. It decays directly to the ground state of the daughter nuclide ^{11}B. From the annihilation of the positron, we get (for each decay) two 511 keV photons emitted in opposite direction. Since the source is injected in the patient, this radiation is emitted from within the patient.

   The half-life of ^{11}C is 20.39 minutes. We can therefore assume that all ^{11}C nuclei will decay in the body of the patient (during *and* after the PET measurement). We know for the activity, A_0, that:

$$A_0 = \lambda \cdot N_0 = \frac{N_0 \cdot ln(2)}{t_{1/2}} \tag{16}$$

   From this we get the number, N_0, of C-11 nuclides (at the time of injection):

$$N_0 = \frac{A_0 t_{1/2}}{ln(2)} = \frac{1.5 \cdot 3.7 \cdot 10^{10} \cdot 60 \cdot 20.39}{ln(2)} = 9.7957 \cdot 10^{13}. \tag{17}$$

   The kinetic energy of the β^+-particle can be converted in different ways (e.g. bremsschtrahlung), but here I assume that the kinetic energy is all converted by the collisions in the body. Assuming an average of 0.5 times the Q-value (1982 keV) for the kinetic energy (since the neutrino takes the other half), we get an absorbed energy of 991 keV in the body for each decay. In addition, the patient will absorb some fraction of the 511 keV photon intensity. Assuming an average of 10 cm from the point of decay to the surface of the body, we can check the absorbed fraction for 511 keV photons:

$$\epsilon_1 = 1 - \frac{I}{I_0} = 1 - e^{-\mu x} = 1 - e^{9.598 \cdot 10^{-2} \cdot 1.06 \cdot 10} = 1 - 0.36154 = \tag{18}$$
$$0.63846, \tag{19}$$

   where the density and abs. coeff. for soft tissue is used. The total effective dose, D, (here it is equal to the absorbed dose (gamma, beta, full body)):

$$D = \frac{N_0(E_{photons}\epsilon_1 + E_{beta}))}{m} = \tag{20}$$
$$\frac{9.7957 \cdot 10^{13}(2 \cdot 511 \cdot 0.63846 + 991) \cdot 10^3 \cdot 1.602 \cdot 10^{-19})}{80} = \tag{21}$$
$$0.32239[Sv], \tag{22}$$

   where the factor 2 comes from the fact that two 511 keV are emitted, and where the body mass is assumed to be 80 kg. It should be noted that this calculated dose of 322 mSv is quite high. For diagnostics of this type it is unusual to have a much higher full body doses than 10 mSv.

   For the nurse, we assume that she is only affected by the 511 keV photons. Since we have estimated above that about 36% of the radiation escapes the patient, we need only to calculate the absoption, ϵ_2 in the nurse. We assume that the nurse is (sitting down) 1.2 meters high, 40 cm wide, and (on average) 25 cm thick. The absorption fraction is now:

$$\epsilon_2 = \frac{I}{I_0} = e^{-\mu x} = e^{-9.598 \cdot 10^{-2} \cdot 1.06 \cdot 25} = 0.078593 \tag{23}$$

Assuming that the nurse sits 2 meter from the patient, we get the solid angle fraction f:

$$f = \frac{1.2 \cdot 0.4}{4\pi \cdot 2^2} = 0.0095493 \tag{24}$$

We know that:

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-ln(2)(t/t_{1/2})} \tag{25}$$

The number of C-11 nuclei in the end of the PET measurements (30 min) is now N(t=30min). The number of decays during 30 minutes, N_{d30}, is therefore:

$$N_{d30} = N_0 - N(30min) = 9.7957 \cdot 10^{13} \cdot (1 - e^{-ln(2)(30/20.39)}) = \tag{26}$$
$$6.2628 \cdot 10^{13} \tag{27}$$

We see that the effective dose for the nurse is now (again , it is equal to the absorbed dose (gamma, beta, full body)):

$$D = \frac{N_{d30}(E_{photons}\epsilon_1\epsilon_2 f)}{m} = \tag{28}$$
$$\frac{6.2628 \cdot 10^{13} \cdot 2 \cdot 511 \cdot \epsilon_1 \cdot \epsilon_2 \cdot f \cdot 10^3 \cdot 1.602 \cdot 10^{-19})}{70} = \tag{29}$$
$$7.0 \cdot 10^{-5} [Sv] \tag{30}$$

where we have assumed 70 kg for the nurse. This dose, 70 μSv is not high in itself, but we should remeber that the nurse might be exposed several times per week.

For the radiation protection, we can then make a wall (for the nurse to sit behind, e.g. 1 meter high, 1 meter wide) at the side of the PET equipment. The thickness, x, of this wall should be enough to reduce the 511 keV photon intensity be a factor of 100:

$$\frac{1}{100} = e^{-\mu x} \tag{31}$$
$$x = \frac{ln(100)}{\mu} = \frac{ln(100)}{0.1614 \cdot 11.35} = 2.5139 [cm], \tag{32}$$

where we have used lead as material. We see that it is enough to make the lead wall about 25 [mm] thick.

2. We start by finding out the activity of the radium-226 present in the body. Here, we might remember that the definition of *1 Ci* is, in fact, the activity

of *one gram* of radium-226 (this can easily be confirmed: $A = \lambda N_A/226$). In the present case, the activity of the decay of radium-226 is therefore:

$$A = 3.7 \cdot 10^4 [Bq]. \tag{33}$$

In the problem it is assumed that this body activity (or amount of radium-226) is constant. We should now calculate the dose to the body.

Radium-226 is part of a decay chain. From the decay of Ra-226 we have: Ra226-α-Rn222-α-Po218-α- Pb214-β^--Bi214-β^--Po214-α- Pb210-β-Bi210-β-Po210-α-Pb206

All the nuclides in the chain after Ra-226 have short half-lives (e.g. much less than one year), until we reach Pb-210. This nuclide has a half-life of over 22.3 years. We assume that the radium have entered the body recently (not over many years) and therefore we can assume that the decay rate from Pb-210 is low in comparison to Ra226. It is then reasonable to assume that the following short lived nuclides will decay in the body, at the same rate as Ra-226: Rn222, Po218, Pb-214, Bi-214, and Po-214.

But we need to consider one more thing. Radon (Rn) is an inert gas. Therefore, it *might* escape the body when it is formed (from the decay of Ra-226), before it decays (with a half-life of 3.8 days) to Po-218). If it does escape, all the other nuclides will decay outside of (and far from) the body, leaving no dose. It is however not *obvious* that radon will escape. Radium will (like calcium) be absorbed in the skeleton, and radon formed inside the bone might not diffuse directly to the body surface.

We first assume that the radon will escape immidiately. Then, we need only to consider the decay from Ra226 to Rn222. Both the alpha particle and the recoil will contribute to the dose, so we can use the Q-value of 4.871 MeV as the energy of each decay. Only 5-6 % will populate excited states. The most intense gamma has a low energy (186 keV), so most of its intensity will be absorbed by the body. We therefore make the approximation of disregarding the populated states and gamma intensity.

The absorbed dose is then:

$$D = \frac{3.7 \cdot 10^4 \cdot 4.871 \cdot 10^6 \cdot 1.602 \cdot 10^{-19} \cdot 365.25 \cdot 3600 \cdot 24}{60} = \tag{34}$$

$$0.015186[Gy], \tag{35}$$

assuming 60 kg for the employee. For alpha radiation, we use a radiation weighing factor of 20, so we get an effective dose of 0.30 [Sv].

If we make the assumption that the radon will always stay in the body so that the decay chains continue in the body, we must consider several decays. The main part of the energies involved is alpha decay energies (Q-values):
From Rn222 to Po218: 5590 keV.
From Po218 to Pb214: 6115 keV.
From Po214 to Pb210: 7833 keV.

The beta energies are lower (using 0.5 times the Q-value):
From Pb214 to Bi214: 511 keV.

From Bi214 to Po214: 1636 keV.

When adding the total energy for alpha and for beta, and also considering the radiation weighting factor (20 for alpha, 1 for beta and gamma), we realise that the beta contribute with less than 1% to the total effective dose. We therefore only consider the alphas.

$$D = \tag{36}$$
$$\frac{3.7 \cdot 10^4 \cdot 20 \cdot (4.871 + 5.590 + 6.115 + 7.833) \cdot 10^6 \cdot 1.602 \cdot 10^{-19} \cdot 365.25 \cdot 3600 \cdot 24}{60} \tag{37}$$
$$= 1.5219[Sv] \tag{38}$$

We see that the dose is about five times higher if we assume that the radon does not escape. The real answer is, of course, somewhere in between the two extreemes, and depends on the details of diffusion rates of heavy inert gases in bone structure (beyond the scope of this course).

A person that wears a watch with a dial containing Ra-226 *will* receive a dose, but a *very low dose*. The alpha particles (representing the major part of the decay energy) will not escape the watch (they will be stopped by the glas or steel). However, some fraction of the gammas emitted from excited states of some of the nuclei in the decay chain, will penetrate the glas or steel. One example is the 186 keV gamma emitted from the excited state of radon-222. If the radon stays in the watch (this depends on the construction of the watch), there are several other gamma decays (involved in the decay chain) to consider.

3. First, we see (from the table of isotopes and/or the nuclide chart) that Co-57 decays by *electron capture*, populating (mainly) the excited state at energy 136 keV in Fe-57. We see that around 11% (12/112) of the intensity decay by emitting a 136 keV gamma, and the rest (89%) by emitting two gammas; 122 keV and 14 keV. From now on, we consider these three gamma photons.

   We assume that Dr.A has the source close to her body. We can first assume that she has it 20 cm in front of her, and that she is is 1.6 meters high, 40 cm wide, and (on average) 25 cm thick, and weighs 60 kg. We also assume that Dr.B has the same body size. In this case it gives *more* than 50% solid angle ratio. But we realise that if the source was placed very close to the body, e.g. at 1 mm distance, then only approximate 50% of the radiation would be directed into the body. So, from that we realise that a rectangular body area divided by a sphere (of 40 cm radius) is not a very good approximation at this close range. On the other hand, a more detailed model is a bit difficult to come up with quickly. If we assume that the solid angle fraction is 50%, then we have assumed the worst-case-scenario of Dr.A holding the source at very short distance. So, this is out assumtion from now on.

   Dr.B, on the other hand stands behind Dr.A, we assume at a disctance of 1.5 meters from the source. Now we use the rectangle/sphere method, and get a solid angle fraction of $1.6 \cdot 0.2/(4 \cdot 1.5^2 \pi) = 0.023$.

For the absorption coefficient (μ/ρ) of soft tissue we have, for the 14 keV energy (we use the value at 15 keV) 1.7 cm^2/g. For the 122 keV and 136 keV lines we use the mean value between 100 keV and 150 keV, e.g. 0.16 cm^2/g.

The transmitted intensity fraction (I/I_0) of the three energies for a 25 cm thick body is:

$$t_{14} = e^{-\mu x} = e^{-1.7\cdot 1.06\cdot 25} = 2.7 \cdot 10^{-20} \quad (39)$$

$$t_{122} = t_{136} = e^{-\mu x} = e^{-0.16\cdot 1.06\cdot 25} = 0.0144, \quad (40)$$

where the density for soft tissue is used. We see immidiately that the 14 keV photons are completely stopped by Dr.A's body (and will not reach Dr.B). For the 122 and 136 keV gammas, only about 1.4% will pass through the body of Dr.A, i.e. 98.56 % will be absorbed.

Now we can calculate the absorbed dose for Dr.A:

$$D = \quad (41)$$

$$\frac{0.5\cdot 200\cdot 10^{-3}\cdot 3.7\cdot 10^{10}\cdot (0.11\cdot 136\cdot 0.9856+0.89\cdot (14+122\cdot 0.9856))\cdot 10^{3}\cdot 1.602\cdot 10^{-19}25}{60} = \quad (42)$$

$$3.3 \cdot 10^-5 [Gy] \quad (43)$$

The effective dose (full body, gamma) for Dr.A is now 0.33 [μSv].

Now we can calculate the dose for Dr.B:

$$D = \quad (44)$$

$$\frac{0.023\cdot 200\cdot 10^{-3}\cdot 3.7\cdot 10^{10}\cdot (0.11\cdot 136\cdot 0.0144+0.89\cdot 122\cdot 0.0144)\cdot 10^{3}\cdot 1.602\cdot 10^{-19}\cdot 25}{60} = \quad (45)$$

$$2.0 \cdot 10^-8 [Gy] \quad (46)$$

The effective dose (full body, gamma) for Dr.B 20 [nSv]. Dr.A shields the radiation, saving Dr.B from receiving a much higher dose.

To conclude, the dose to Dr. A is quite low (comparable to an hour of background radiation), and Dr. B receives an extremely low dose from the insident, thanks to the shielding of Dr.A.

4. A detector suitable for detecting the gamma energies after the activation of sodium in the body is a *germanium detector*. Thanks to its extremely good *energy resolution* (0.2% at 1 MeV), we are able to separate the peaks of interest from other radiation, e.g. background gamma radiation.

   When neutrons are absorbed in the sodium (Na-23) of the body, Na-24 is formed. It then decays by β^- (half-life 14.96 hours), and populates excited states in Mg-24, mainly (99.944%) the 4+ state at 4123 keV. Two gamma energies that we expect to see (as photo peaks) in the energy spectrum is 2754 keV and 1369 keV, since they are emitted when the 4+ state decays to the ground state.

   We expect two people to die from the accident (around 4 Sv, one has a 50% chance of survival, it is unlikely to survive 18 Sv or 10 Sv). In addition to these two (that of course suffer from radiation sickness before they die), we expect one more employee (2.5 Sv) to suffer from radiation sickness but survive. Below around 1 Sv, we do not expect any radiation sickness.

This is also exactly what happened in the real accident in Tokaimura. Three people developed symptoms of radiation expose shortly after the accident (sickness, etc). One of the three survived (2.5 Sv). The other two died, in spite of intense medical treatment.