Solutions to the Written Exam, Radiation Protection, Dosimetry, and Detectors (SH2603), October 25, 2008

Section A

1. The nuclide ^{60}Co decays to ^{60}Ni by β^--decay. The Q-value of the beta decay is 2823.9 keV, and this energy (minus a very small recoil energy for the daughter nucleus) is shared between the emitted electron and the anti-neutrino. The daughter nuclide ^{60}Ni is populated (in 99.925 % of the decays) in an excited state, with an energy (above the ground state) of 2506 keV. The nucleus then decays to its ground state by emitting two gamma photons, one with the energy 1173 keV, and the other with 1333 keV.

2. A linear energy calibration can be expressed like this: $E = a \cdot chn + b$, where a and b are constants, E is the energy, and chn is the channel number at E. For ^{22}Na, we have the two energies: $E_1 = 511$keV (we get two 511 keV photons from the annihilation of an atomic electron and the positron from the beta-plus decay of Sodium-22) and $E_2 = 1275$keV at the channel numbers $chn_1 = 823$, and $chn_2 = 2086$. This gives us $a = 0.60491$ and $b = 13.16$. The unknown peak at channel number 1073 therefore corresponds to the energy 662 keV.

3. Work involving ionising radiation should be *justified*.

4. Acute radiation sickness appears after the person receives a dose above around 1 Sv.

5. Potassium-40 (^{40}K) and carbon-14 (^{14}C).

6. From the table of absorption coefficients we see that μ_m for lead at 60 keV is 5.021 g/cm^2:

$$I = I_0 \cdot e^{-\mu x} = I_0 \cdot e^{-\mu_m \rho x} = 100000 \cdot e^{-5.02111.340.1} = 336.67 \quad (1)$$

   The new intensity will be 337 photons per second.

7. The three ways for gamma photons to interact with materials are *photoelectric effect*, *Compton scattering*, and (if the photon energy is above 1022 keV) *pair production*. All three processes are possible in this case.

8. Assuming the constant density of the nuclear matter we get for the nuclear radius R:

$$R = R_0 \cdot A^{1/3} \quad (2)$$

   Here, R_0 is a constant, often set to $1.2 \cdot 10^{-15}$ m. With $A = 238$ (the most abundant uranium isotope is ^{238}U) we get a radius of $7.4 \cdot 10^{-15}$ m.

9. The radioactive gas that is the origin of ^{214}Bi in buildings is radon-222 (^{222}Rn). The decay chain from ^{222}Rn to ^{214}Bi is:
   Rn222-(alpha)-Po218-(alpha)-Pb214-(beta-)-Bi214
   Bi214 is populated in excited states that decay by gamma emission.

10. For the range of electrons, we use Katz & Penfold. At an energy of 1 MeV it will give us a range of 0.412 grams per cm^2. Dividing with the density of air ($1.205 \cdot 10^{-3}$ g/cm2) gives a range of 3.4 m.

    For gamma, the value of μ_m for air at 1 MeV is $6.358 \cdot 10^{-2}$ cm2/g. The distance of half intensity means that we have:

$$\frac{1}{2} = e^{-\mu_m \rho x} \tag{3}$$

$$x = \frac{ln(2)}{\mu_m \rho} = 9047[cm] \tag{4}$$

    So, the 1 MeV gamma range of half intensity is about 90 meters.

Section B

1. The density ρ of argon and the gamma absorption coefficient μ_m of argon at 1 MeV are found in the tables. The number N of gamma that interacts with the gas is:

$$N = I_0 - I = I_0 - I_0 \cdot e^{-\mu_m \rho x} = 1000 - 1000 \cdot e^{-0.05762 \cdot 0.001662 \cdot 10} = 0.95719 \tag{5}$$

   We see that (on average) only about 1 gamma photon will be detected, out of 1000 photons.

   If we put a thin lead sheet in front of the detector, the count rate in the detector will *increase*. The reason is that some of the gamma photons will now interact, (by Compton scattering, or photo effect) with the lead. The photo- and Compton electrons will have high energies (photo:1 MeV, Compton: less, but still in the MeV range). They can penetrate the lead and enter the ionising chamber, if the gamma/lead interaction takes place where the remaining thickness of the lead is smaller than the range of the electrons. If the electrons reach the gas, they will always give a signal, since they slow down in the gas, transferring energy.

   To estimate the new efficiency, we first calculate the range R of 1 MeV electrons in lead, using the Katz & Penfold formula:

$$R[g/cm^2] = 0.412 \cdot E^{1.265 - 0.0954 \cdot lnE} \tag{6}$$

   In this formula, E should be expressed in MeV. This gives $R = 1.4598[g/cm^2]$. With $\rho_{lead} = 11.34[g/cm^3]$ we get a range of 0.0363 cm. From this, we see that it is only the electrons emitted from the inner last 0.363 mm before the gas that could be detected.

   We have a μ_m of $7.102 \cdot 10^{-2}$ for 1 MeV gamma in lead. First, let us calculate how many photons that will pass through the first (1-0.363=0.637) [mm] unaffected. we get:

$$I^{new} = I_0 \cdot e^{-\mu_m \rho x} = 1000 \cdot e^{-0.07102 \cdot 11.34 \cdot 0.0637} = 995 \tag{7}$$

   In the last part of the lead (0.363 mm), a number N_i of the gamma photons will interact with electrons. In the equation below, I_0^{new} is the same (995) as I^{new} in the previous equation.

$$N_i = I_0^{new} - I_0^{new} \cdot e^{-\mu_m \rho x} = 995 - 995 \cdot e^{-0.07102 \cdot 11.34 \cdot 0.0363} = 29 \tag{8}$$

So, 29 gamma photons interact with electrons in the lead. Assuming that all electrons enter the gas, we have increased the detection efficiency from 1 photon per 1000 to 30 photons per 1000 (i.e. from 0.1% to 3%). Due to the angular distribution of the Compton scattering, etc, the problem is more complicated, but our result, using these simple assumptions, is quite realistic.

2. We have two gamma photons emitted from Co-60 (1173 keV and 1333 keV). We start by calculating the solid angle fraction f_{sa} of the nurse:

$$f_{sa} = \frac{1.65 \cdot 0.3}{4 \cdot \pi \cdot 3^2} = 0.0043768 \tag{9}$$

Assuming 20 cm of thickness x_n for the nurse, the fraction f_{abs} that gets absorbed in the nurse (we can use μ_m for tissue at 1.25 MeV = $6.265 \cdot 10^{-2}$ cm2/g) is now:

$$f_{abs} = 1 - e^{-\mu_n \rho x_n} = 1 - e^{-0.06265 \cdot 1 \cdot 20} = 0.71435 \tag{10}$$

The effective dose D over the time t of 8 hours is now:

$$\begin{aligned} D &= \frac{A \cdot E_\gamma \cdot f_{sa} \cdot f_{abs} \cdot t}{m} = & (11)\\ &= \frac{5 \cdot 3.7 \cdot 10^{10} \cdot (1173 + 1333) \cdot 10^3 \cdot 1.602 \cdot 10^{-19} \cdot 0.0043768 \cdot 0.7143 \cdot 3600 \cdot 8}{60} & (12)\\ &= 0.11145, & (13) \end{aligned}$$

where we have assumed a mass for the nurse of 60 kg. We get a dose of 0.11 Sv.

To reduce the dose to $1 \mu Sv$ we need to reduce the intensity to a fraction F of the original intensity:

$$F = \frac{1 \cdot 10^{-6} Sv}{0.11145 Sv} = 8.97 \cdot 10^{-6} \tag{14}$$

I choose lead as the shielding material. We see that μ_m for lead at 1.25 MeV is $5.876 \cdot 10^{-2}$ cm2/g. We get:

$$\frac{I}{I_0} = 8.97 \cdot 10^{-6} = e^{-\mu_m \rho x} \tag{15}$$

Solving for x, we get:

$$x = \frac{ln(8.97 \cdot 10^{-6})}{-\mu_m \rho} = 17.4[cm] \tag{16}$$

We need at least 17.4 cm of lead to shield the source. A possible design would be a cylinder made of lead, with diameter 40 cm and height 40 cm, and with the source in a small hole in the centre. The weight would then be around 570 kg.

3. There are (at least) three ways to identify Po-210 with detectors of ionising radiation. One possibility is to measure the energy of the alpha

radiation with a well calibrated alpha detector (e.g. a surface barrier silicon detector). The alpha energy of Po-210 should be 5.305 MeV. Another possibility is to measure the decay half-life (using the alpha decay) carefully over a number of days, to see that the half-life is in fact 138.38 days. A third possibility is to measure the very weak gamma ray of 803 keV emitted from an excited state of the daughter nucleus Pb-206. A germanium detector should be used for this, and it is only possible if the source is strong enough (as in this case).

10 micrograms of Po-210 corresponds to a number of atoms N:

$$N = \frac{m \cdot N_A}{m_u} = \frac{10 \cdot 10^{-6} 6.022 \cdot 10^{23}}{209.98} = 2.8679 \cdot 10^{16} \tag{17}$$

The activity A can now be calculated:

$$A = N \cdot \lambda = N \cdot \frac{ln(2)}{T_{1/2}} = 2.8679 \cdot 10^{16} \frac{0.69315}{138.376 \cdot 24 \cdot 3600} = 1.6627 \cdot 10^{9} [Bq] \tag{18}$$

This corresponds to about 45 mCi.

We can assume that The absorbed dose D is now calculated with the knowledge of E_α, the alpha energy release (Q-value) of 0.000 MeV:

$$D = \frac{A \cdot E_\alpha \cdot t}{m_{body}} = \frac{1.6627 \cdot 10^{9} \cdot 5.407 \cdot 10^{6} \cdot 1.602 \cdot 10^{-19} \cdot 24 \cdot 3600}{75} = 1.6591 [Gy] \tag{19}$$

To get the effective dose, we should multiply by 20 since we deal with alpha radiation. The effective dose is then around 33 Sv. This is a lethal dose.

4. Silver has two stable isotopes, ^{107}Ag, an ^{109}Ag. By absorbing a neutron they will transform into ^{108}Ag, an ^{110}Ag. Both these nuclides are radioactive by β^--decay. Silver-108 has a half-life of 2.38 minutes and decays to Cadmium-108. Silver-110 has a half-life of 24.6 seconds and decays to Cadmium-110. Other decay channels exist, but with less probability.

   Both nuclides emit beta electrons, and they also populate excited states in their daughter nuclides. Therefore, we expect to see e.g. the gamma photons of energy 633 keV (internal transition in Cd-108) and 658 keV (Cd-110).