Solutions to the Written Exam, Radiation Protection, Dosimetry, and Detectors (SH2603), June 10, 2008

Section A

1. The electron capture competes with β^+-decay for nuclides with a proton excess. The electron capture process can be written:

$$p + e^- \rightarrow n + \nu_e \quad (1)$$

Here, p is a proton in the mother nucleus. n is a neutron in the daughter nucleus, ν_e is an electro-neutrino that is send out from the nucleus, and e^- is an electron that is captured. The origin of the electron is in one of the *atomic* electron orbitals. It is captured by the nucleus, thereby creating a hole in an atomic shell. If one of the inner shell electrons are captured by the nucleus in a heavy nucleus, the hole will be filled by one of the other electrons. At the same time a *characteristic X-ray* is emitted (secondary radiation).

2. We use the formula for absorption of gamma photons: $I = I_0 exp(-\mu x)$. The gamma photons emitted from ^{40}K have the energy 1461 keV. At this energy we get (from the table) approximately $\mu/\rho = 4.9 \cdot 10^{-2}$ [cm^2/g]. The density, ρ, for iron is 7.874 [g/cm^3]. Setting $I = I_0/5$ and solving for x, we get:

$$x = \frac{ln(I/I_0)}{-\mu} = \frac{ln(1/5)}{-4.9 \cdot 10^{-2} \cdot 7.874} = 4.2[cm] \quad (2)$$

The iron plates should be 4.2 cm thick.

3. Exposed to a flux of thermal neutrons, a ^{238}U nuclei can absorb a neutron, thereby creating the nuclide ^{239}U. This nuclide is unstable to β^--decay, with a half-life of 23.5 minutes. The daughter nucleus in the decay is ^{239}Np, which is also unstable to β^--decay, in this case with a half-life of 2.3 days. The daughter nucleus of ^{239}Np is the plutonium isotope ^{239}Pu, (which is unstable to alpha decay (and spontaneous fission), with a long (> 24000 years) half-life). In this way, plutonium is produced by uranium-238.

4. To calculate the time since the death of the mammoth, we use the decay formula: $N = N_0 exp(-\lambda t)$. We note that λ can be re-written with the half-life $t_{1/2}$ (which is 5730 years) as $\lambda = ln(2)/t_{1/2}$. One gram of ^{12}C contains $N_A/12$ atoms. Therefore, one gram of natural carbon contains approximately $10^{-12} N_A/12$ atoms of the ^{14}C isotope. Here, we have neglected the 1% C-13 abundance in natural uranium. Solving the above formula for the time t (in years), we get:

$$t = -t_{1/2}\frac{ln(N/N_0)}{ln(2)} = -5730\frac{ln(3.945 \cdot 10^8/(\frac{1}{12}6.023 \cdot 10^{23} \cdot 10^{-12}))}{ln(2)} = \quad (3)$$

$$= 40060 \quad (4)$$

We see that this particular mammoth died about 40000 years ago.

5. High-energy electrons will loose energy in two processes. The first process can be seen as a *collision* process. The electric charge of the electrons interact with the charged particles in the bulk matter, transferring part of its kinetic energy to (mostly) other electrons. The second, competing process can be seen as a *radiation* process. when the electrons interacts with charged ions in the matter, they experience strong changes in direction of the velocity vector, i.e. strong acceleration. An accelerated charged particle will emit radiation. This radiation is emitted in the form of photons, and is called *Bremsstrahlung.* We can write the stopping power for electrons like this:

$$\left(\frac{dE}{dx}\right)_{total} = \left(\frac{dE}{dx}\right)_{collision} + \left(\frac{dE}{dx}\right)_{radiation} \tag{5}$$

   The radiation process gets more important for higher energies.

6. The sequence of yearly doses is not allowed for more than one reason. First, during the five years 1995-1999 the employee receives a total accumulated dose of 5+29+27+19+24=104 mSv. He therefore break the 5-year limit of 100 mSv already in 1999. He should have been stopped at that time. (subsequent 5-year periods also break the 100 mSv limit. In addition, the yearly dose (50 mSv) is broken 2001 and 2003).

7. The radioisotope ^{32}P is unstable to β^--decay. There is no gamma decay associated with this decay. The first priority is therefore to stop the electrons. Estimating/calculating the range and choosing the right thickness will therefore stop the beta electrons. But the beta electrons have a high energy (up to 1.7 MeV in this case), and can easily kick out electrons from the inner (atomic) shells in the material (tin, in this case). When the electron holes are filled, *characteristic X-rays* will be emitted. The radiation detected in the GM-counter correspond to these X-rays. The X-rays can penetrate the thin layer of tin (easier than the electrons) and more radiation protection must be used to stop also them, and not only the beta electrons.

8. Simplifying, we assume that the teacher has a rectangular cross section, with a width of 0.4 m and a height of 1.8 m. The thickness is assumed to be 0.25 m, and we set the mass of the teacher to 80 kg. The fraction of the solid angle f_s:

$$f_s = \frac{0.4 \cdot 1.8}{4\pi 5^2} = 0.00229 \tag{6}$$

   The energy of the gamma rays from a Cs-137 source is 662 keV. The fraction of absorption in the soft tissue of the body $((\mu/\rho)_{tissue}$ (using the value for 600 keV) from table):

$$f_{abs} = (1 - e^{-\mu x}) = 1 - e^{-8.87 \cdot 10^{-2} \cdot 1.06 \cdot 25} = 0.905 \tag{7}$$

   The absorbed dose, D, is then:

$$D = \frac{10 \cdot 10^{-6} \cdot 3.7 \cdot 10^{10} \cdot 662 \cdot 1000 \cdot 1.602 \cdot 10^{-19} \cdot f_s \cdot f_{abs} \cdot 300 \cdot 3600}{80} = \tag{8}$$

$$= 1.0978[\mu G] \tag{9}$$

In this case (gamma, full-body dose), the absorbed dose is the same as the effective dose. The teacher gets an effective dose of 1.1 μSv each year from the work in the student laboratory.

9. The ionizing chamber detector is filled with a gas (I assume here, for the sake of discussion, that the gas is argon). For an alpha particle to enter the gas it must first penetrate a window in the detector wall, (typically made of a thin sheet of metal). When the alpha particle enters the gas, it will interact with the charges in the gas. The alpha particle will slow down, transferring energy (mainly) to the electrons in the argon atoms, kicking the electrons away from the atoms. In this process, many free electrons (and positively charged argon ions) will be produced. An electric field is present in the detector. If the detector is of cylindrical shape, this field is created by putting a high voltage (typically a few hundred Volts) on a thin rod or wire along the centre axis of the cylinder. The cylindrical shell is connected to zero potential. In the electric field, the free electrons will start to accelerate towards the centre rod. If the voltage is high enough, the accelerated electrons will kick out new electrons, creating an avalanche effect. The free electrons will finally (a very short time after the alpha particle appeared) reach the centre rod, thereby creating a charge pulse. An electric circuit (usually decoupled with a capacitor) is connected to the central rod, so that the electric pulse signal can be amplified.

10. The natural abundance of the radioactive isotope potassium-40 is 0.0117%. The atomic weight m_u of K-40 is 39.96 u. The number of ^{40}K atoms in 451 mg of natural potassium is then:

$$N = \frac{0.0117 \cdot 10^{-2} \cdot 451 \cdot 10^{-3} \cdot N_A}{m_u} = 7.95 \cdot 10^{17} \tag{10}$$

The half-life is $1.29 \cdot 10^9$ years, which means that $\lambda = ln(2)/t_{1/2}$ is equal to $1.703 \cdot 10^{-17}[s^{-1}]$. We now have for the activity, A:

$$A = \lambda \cdot N = 13.54 \tag{11}$$

We see that we have an activity of about 14 Bq for the banana.

Section B

1. We assume that a typical child has a cross section of 1.2 meter high and 0.3 meter wide, and an average thickness of 0.15 meter.

   Cs-137 emit beta electrons, and 661 keV gamma photons. This is an open source, so we must consider the electrons.

   Starting with the electrons, there are two different contributions. First, 94.4% of the electrons populate the excited (662 keV) level in Ba-137. Therefore their energy has a maximum at $Q_\beta - 662 = 1176 - 662 = 514$ keV.

   the Katz and Penfold formula gives us, using $\rho_{air} = 1.205 \cdot 10^{-3}[g/cm^3]$ a range of $(1/1.205 \cdot 10^{-3}) \cdot 0.412 \cdot 0.514^{1.265-0.0954 \cdot ln(0.514)} = 141$ cm. For the (5.6%) beta electrons with the maximum energy of 1176 keV, we get in the same way: $(1/1.205 \cdot 10^{-3}) \cdot 0.412 \cdot 1.176^{1.265-0.0954 \cdot ln(1.176)} = 419$

cm. We see that the electrons are indeed stopped by the air, after just a few meters. They should therefore not contribute to the dose.

For the 662 keV gamma rays, we need to consider the air absorption, due to the long distance. We have:

$$f_{air} = \frac{I}{I_0} = e^{-\mu \cdot x} = e^{-8.055 \cdot 10^{-2} \cdot 1.205 \cdot 10^{-3} \cdot 80 \cdot 10^2} = 0.46, \qquad (12)$$

where we have used the absorption coefficient for 600 keV gammas in air. We see that only 46% of the radiation reaches the day-care centre. For the absorption of the walls, we have, in the same way:

$$f_{concrete} = \frac{I}{I_0} = e^{-\mu \cdot x} = e^{-8.236 \cdot 10^{-2} \cdot 2.3 \cdot 10} = 0.15 \qquad (13)$$

We see that only 15% passes the walls. We also need the fraction that interacts with the tissue:

$$f_{tissue} = 1 - \frac{I}{I_0} = 1 - e^{-\mu \cdot x} = 1 - e^{-8.873 \cdot 10^{-2} \cdot 1.06 \cdot 15} = 0.76 \qquad (14)$$

The solid angle fraction is:

$$f_{s.a.} = \frac{1.2 \cdot 0.3}{4\pi 80^2} = 4.476 \cdot 10^{-6} \qquad (15)$$

The Energy released per second from the source, in the form of gamma photon (662 keV) is:

$$P = 662 \cdot 10^3 \cdot 100 \cdot 3.7 \cdot 10^{10} \cdot 1.602 \cdot 10^{-19} \cdot 0.944 = 3.70 \cdot 10^{-1} [J/s] \quad (16)$$

Here, note the 0.944 factor. Only this fraction in the decay of Cs-137 populates the 662 keV state in Ba-137.

Assuming a mass of 25 kg for one child, we have for the absorbed dose:

$$D = \frac{P f_{tissue} f_{s.a.} f_{air} \cdot 3600(5 f_{concrete} + 3)}{25} = 3.13 \cdot 10^{-4} [G] \qquad (17)$$

For gamma, and full body dose, we therefore have an effective dose of 0.31 [mSv]. This is a very small dose, comparable with dose we on average receive from the background radiation over one month. The children should therefore be safe (no radiation sickness, no long-term effects).

2. We start by calculating the energy release per second from the source:

$$P = (1173 + 1332) \cdot 10^3 \cdot 1000 \cdot 3.7 \cdot 10^{10} \cdot 1.602 \cdot 10^{-19} = 14.85 [J/s] \quad (18)$$

This is a considerable energy release rate. The radiation protection will heat a bit due to this, although the 15W power should easily be cooled by the surrounding air.

Assuming height/width/thickness of a person to 1.7m/0.4m/0.25m, we have, for 3 meters distance a solid angle fraction of:

$$f_{s.a.} = \frac{1.7 \cdot 0.4}{4\pi 3^2} = 0.006 \qquad (19)$$

A person working in the floor below (say 3 meters below), will have a different cross section (and thickness), but it depends strongly on the angle, etc, so to simplify, we use the same solid angle fraction for that case.

The absorption in the 10 cm of concrete is:

$$f_{concrete} = \frac{I}{I_0} = e^{-\mu \cdot x} = e^{-5.807 \cdot 10^{-2} \cdot 2.3 \cdot 10} = 0.26, \tag{20}$$

where we have used the absorption coefficient at 1.25 MeV in concrete.

Effective dose is equal to absorbed dose here (gamma, full body). Now, we calculate the absorbed dose (assuming a 70 kg person) (in the two cases), without protection:

$$D_{below} = \frac{P f_{concrete} f_{s.a.} \cdot 3600 \cdot 8 \cdot 200}{70} = 1932[Sv/year] \tag{21}$$

$$D_{inroom} = \frac{P f_{s.a.} \cdot 3600}{70} = 4.59[Sv/hour] \tag{22}$$

We must reduce the first dose (floor below) by a factor of: $1932/(0.1 \cdot 10^{-3}) = 1.932 \cdot 10^7$. We must reduce the second dose (same room) by a factor of: $4.59/(10 \cdot 10^{-6}) = 4.59 \cdot 10^5$.

Assuming no absorption in the air, and selecting lead as material, we can start by calculating the needed thickness between the source and the person. Using

$$I = I_0 e^{-\mu x}, \tag{23}$$

we have, for the first case (floor below):

$$x = \frac{-ln(I/I_0)}{\mu} = \frac{-ln(1/(1.906 \cdot 10^7))}{5.876 \cdot 10^{-2} \cdot 11.35} = 25.2[cm] \tag{24}$$

we have, for the second case (same room):

$$x = \frac{-ln(I/I_0)}{\mu} = \frac{-ln(1/(4.58 \cdot 10^5))}{5.876 \cdot 10^{-2} \cdot 11.35} = 19.5[cm] \tag{25}$$

So, we need to make a radiation protection with at least 25.2 cm lead *below* the source (to protect the floor below), and at least 19.5 cm *to the side of* the source (to protect people in the same room). There should also be material *above* the source, for obvious reasons. Many designs are possible. Here I choose a cylindrical design. The cylinder is standing up on one of its flat surfaces. The source (assuming that the source itself is very small) is placed in the centre of the cylinder, which is made of lead. The height of the cylinder is 52 cm (2·26 cm), the radius 20 cm. The mass, m, of such a cylinder is:

$$m = V \cdot \rho = \pi r^2 h \rho = \pi \cdot 20^2 \cdot 52 \cdot 11.35 \cdot 10^{-3} = 742[kg] \tag{26}$$

The design is, as we see, lighter than the 1000 kg limit.

3. The Q_α correspond to the full energy release in the alpha decay. The main part of the energy is given as kinetic energy to the alpha particle, but in order to preserve momentum, there must also be a recoil energy given as kinetic energy to the daughter nucleus in the alpha decay. We get:

$$p_\alpha = p_{daughter} \tag{27}$$
$$m_\alpha \cdot v_\alpha = m_{daughter} \cdot v_{daughter} \tag{28}$$
$$v_{daughter} = \frac{m_\alpha}{m_{daughter}} v_\alpha \tag{29}$$

Now, writing the total kinetic energy, e.g. the Q_α-value:

$$Q_\alpha = \frac{m_\alpha v_\alpha^2}{2} + \frac{m_{daughter} v_{daughter}^2}{2} = \tag{30}$$
$$E_\alpha + \frac{m_{daughter}(\frac{m_\alpha}{m_{daughter}} v_\alpha)^2}{2} = \tag{31}$$
$$E_\alpha + \frac{m_\alpha}{m_{daughter}} \frac{m_\alpha v_\alpha^2}{2} = \tag{32}$$
$$E_\alpha (1 + \frac{m_\alpha}{m_{daughter}}) \tag{33}$$

For the case of Po-210, we have $m_\alpha / m_{daughter} = 4/206$. Using the value for E_α from the nuclide chart (5304.4 keV), we get Q_α =5407.4 keV.

For the next part, we use Weiszäcker's formula to calculate the binding energy, E_B, for the three nuclei Po-210, Pb-206, and He-4. Putting the numbers in, we get:

$$E_B(Po210) = 1624.3355[MeV] \tag{34}$$
$$E_B(Pb206) = 1600.9537[MeV] \tag{35}$$
$$E_B(He4) = 30.780[MeV] \tag{36}$$
$$\Delta E_B = 1600.9537 + 30.780 - 1624.3355 = 7.3982[MeV] \tag{37}$$

The difference in the total binding energy, before and after the decay, is about 7.4 MeV. This is in the right order of magnitude, but not very precise. The limitation of the semi-empirical mass formula is evident here. The problem lies in the fact that the formula cannot reproduce the very strong binding energy of the alpha particle which is much larger than for its neighbouring nuclides.

Using experimental masses instead, we get:

$$m(Po-210) = 209.982857[u] \tag{38}$$
$$m(Pb-206) = 205.974449[u] \tag{39}$$
$$m(He-4) = 4.002603[u] \tag{40}$$
$$\Delta M = 209.982857 - (205.974449 + 4.002603) = 0.005805[u] \tag{41}$$

The difference in binding energy corresponds to the difference in mass. Using that 1[u]=931.502 MeV/c^2, the difference in binding energy, now becomes: E=mc^2=0.0058050*931.502=5.407 [MeV]

This is the same energy as the Q-value from the first part.

4. The only stable holmium isotope is ^{165}Ho. The neutron irradiation will make some ^{165}Ho nuclei absorb a neutron to form ^{166}Ho. There are two long-lived states in ^{166}Ho that can be populated in this process. First, we have the ground state (spin=0), with a half-life of about 27 hours. Then, we have a long-lived (isomer) state at spin=7. The half-life for this isomeric state is about 1200 years. (In the neutron absorption, the two states will not be populated equally, but this is a minor effect. If we assume that the two states are equally populated, the answer to this problem will be the same.) The ^{166}Ho nuclei will (from both these states) beta decay to ^{166}Er. Assuming that we have N ^{166}Ho nuclei in the ground state and N ^{166}Ho nuclei in the excited (spin=7) state, the beta activity from the ground state will be several orders of magnitude larger. This is due to the different half-lives. We have for the activity A:

$$A = \lambda N = \frac{ln(2)}{t_{1/2}} \cdot N \tag{42}$$

If we calculate the two beta activities, we see that the activity from the ground state is more than 2.5 million times stronger than the activity from the excited state.

Therefore (looking in the table of isotopes), for the first measurement, we see that the most intense gamma radiation at the energy 80.6 keV, since this is the most populated excited state in ^{166}Er in the beta decay from the ground state of Ho-166.

After one year, all the ground state Ho-166 nuclei have decayed. The only beta activity left is the one from the isomeric state. This beta decay populate several excited states in Er-166. Several gamma rays will be emitted by Er-166 in the internal transitions, following the beta decays. The same gamma energy as before (80.6 keV) *will* indeed be seen in this second measurement, since the 80.6 keV level in Er-166 is populated by gamma decay from states of higher energy. A number of additional gamma energies, that we expect to see clearly in the second experiment are: 711.7 keV, 810.2 keV, 752.2 keV, 184.4 keV, and 280.5 keV. There are many others, most of them with lower intensity.