Solutions to the Written Exam, Radiation Protection, Dosimetry, and Detectors (SH2603), January 14, 2008

Section A

1. The energy release E is the total difference in mass times c^2:

$$E = \left[m(^{235}U) + m_n - \left((3 \cdot m_n + m(^{93}Rb) + m(^{140}Cs)\right)\right] \cdot c^2 \qquad (1)$$

   We then look up the atomic masses in the mass table:
   m(^{235}U)=235.0439231 u, m(^{140}Cs)=139.917277 u, m(^{93}Rb)=92.922033 u, m_n=1.00866501 u
   We then note the conversion factor in the formula sheet: $1u = 931.502 MeV/c^2$, and finally get the value for the energy release: 174 MeV.

2. The ^{90}Sr+^{90}Y source emits electrons from both nuclides. Checking the *table of isotopes* we see that the *continous* energy of the electrons emitted from ^{90}Sr have a maximum (corresponding to the Q-value) of 546 keV. The electrons emitted from ^{90}Y can have a higher energy; up to 2282 keV. So, the most energetic electrons emitted from the source have the energy 2282 keV. Using the Katz & Penfold formula:

$$R[g/cm^2] = 0.412 \cdot E^{1.265 - 0.0954 \cdot lnE} \qquad (2)$$

   In this formula, E should be expressed in MeV. This gives $R = 1.1[g/cm^2]$. With $\rho_{water} = 1[g/cm^3]$ we get a range of 1.1 cm.

3. The continous distribution in the electron energy spectrum corresponds to the beta-decay. The sharp peak in the spectrum reveals that *internal conversion* is involved in the decay process. Internal conversion means that a nucleus de-excites from an excited state to a lower energy state (in this case, from the 279 keV state in the nuclide ^{203}Tl to its ground state), not by emitting a gamma photon, but by giving the excess energy to one of the atomic electrons surrounding the nucleus. The energy of this electron is discrete, but with an energy somewhat lower than the energy difference between the nuclear states (here 279 keV) since the binding energy of the atomic electron must be subtracted.

4. Yes, there will be photons emitted. The nuclide ^{18}F decays by β^+-decay (red colour in the nuclide chart). In the *table of nuclides* we see that the beta decay goes directly to the ground state in ^{18}O, so we do not expect any gamma emission from that nuclide. But the emitted *positron* will quickly slow down in the material surrounding the source and then annihilate with an electron. This will typically create two annihilation photons emitted in opposite directions. Each photon have an energy of 511 keV (half the rest mass of the electron-positron system).

5. In a laboratory, a strong Co-60 source (100 mCi) is kept inside a safe-box with lead walls (5 cm thickness) to limit the gamma radiation exposure of the staff working in the lab. But when the dosimeters are checked it turns out that the dose is still too high. It is decided to reduce the gamma

intensity from the box by a factor of 100. The solution is to increase the lead wall thickness of the safe-box. How thick must the new walls be? [1 p]

We have before and after the modification of the box:

$$I_{old} = I_0 e^{-\mu \cdot x_{old}} \tag{3}$$
$$I_{new} = I_0 e^{-\mu \cdot x_{new}} \tag{4}$$
$$\tag{5}$$

We must increase the absorption by a factor of 100, so that $I_{new} = I_{old}/100$:

$$100 = \frac{I_{old}}{I_{new}} = \frac{e^{-\mu \cdot x_{old}}}{e^{-\mu \cdot x_{new}}} \tag{6}$$
$$ln(100) = \mu \cdot x_{new} - \mu \cdot x_{old} \tag{7}$$
$$x_{new} = x_{old} + \frac{ln(100)}{\mu} = 5 + \frac{ln(100)}{5.876 \cdot 10^{-2} \cdot 11.34} = 11.91[cm] \tag{8}$$

Here, we have taken the mass absorption coefficient for lead at 1.25 MeV (just inbetween the gamma energies of Co-60: 1332 keV and 1173 keV), and the density, from the tables. We see that we need new lead walls with a thickness of almost 12 cm.

6. The continous distribution of energies in the gamma spectrum corresponds to gamma photons that scatter by *Compton scattering* in the detector. In the Compton process, an electron receives part of the photon energy. The photon energy is reduced and the photon changes direction. If the photon then travels *out from the detector* the energy is lost, and the detected energy is less that the photo peak energy. The energy loss is different every time, and therefore the distribution is continous.

7. To get the number N of atoms in 100 tons of pure ^{210}Po, we use the atomic mass m(^{210}Po), and Avogadro's number N_A:

$$N = \frac{m[grams] \cdot N_A}{m_u} = \frac{10 \cdot 10^8 6.022 \cdot 10^{23}}{209.98} = 2.8684 \cdot 10^{29} \tag{9}$$

The activity is calculated using the half-life ($t_{1/2}$) of polonium-210 (138.38 days):

$$A = -\frac{dN}{dt} = -\lambda \cdot N = \frac{ln(2)}{t_{1/2}} N = \frac{0.69315}{138.38 \cdot 24 \cdot 3600} 2.8684 \cdot 10^{29} = \tag{10}$$
$$1.6629 \cdot 10^{22}[Bq] \tag{11}$$

We finally get the power P by multiplying with the energy of the alpha decay:

$$P = A \cdot Q_{alpha} = 1.6629 \cdot 10^{22} \cdot 5.407 \cdot 10^6 \cdot 1.602 \cdot 10^{-19} = 1.44 \cdot 10^{10}[W] \tag{12}$$

We see that we get a power of 14.4 GW thermal power. This is around four times the thermal power of a large nuclear power plant!

8. The difference between *absorbed dose* D and *dose equivalent* H is that the absorbed dose is just the absorbed radiation energy per unit mass of material (unit Gy=J/kg), while the dose equivalent (unit Sv) also depends on the type of radiation involved. The two are related by a weighting factor W_R like this: $H = D \cdot W_R$.

9. We get, for the dose D, time t, energy E, weighting factor W_R, and mass m:

$$D = \frac{A \cdot t \cdot W_R \cdot E}{m} = \frac{10^{-6} \cdot 3.7 \cdot 10^{10} \cdot 24 \cdot 3600 \cdot 20 \cdot 5.638 \cdot 10^{6} \cdot 1.602 \cdot 10^{-19}}{30} = \quad (13)$$

$$0.0019[Sv] \quad (14)$$

   We see that the dose to the child is close to 2 mSv.

10. 50mSv

Section B

1. The potassium we have in our bodies consists of (checking the abondances e.g. on the nuclide chart) 93% K-39, 7% K-41, and 0.0117% K-40. We note from the nucide chart, and the table of isotopes that ^{40}K is radioactive, so this is the nuclide of interest here. K-40 has an atomic mass m_u of 39.96 u, so the number N of K-40 nuclei in the body with mass 70 kg is (with 2 grams of potassium per kilogram body weight) calculated by:

$$N = \frac{0.0117 \cdot 10^{-2} \cdot 140 \cdot N_A}{m_u} \quad (15)$$

   where N_A is Avogadros's number. The activity A for a 70 kg body is then:

$$A = \lambda N = \frac{1.17 \cdot 10^{-4} \cdot ln(2) \cdot 140 N_A}{t_{1/2} m_u} = \quad (16)$$

$$\frac{ln(2) \cdot 1.17 \cdot 10^{-4} \cdot 140 \cdot 6.023 \cdot 10^{23}}{1.28 \cdot 10^{9} \cdot 365.25 \cdot 24 \cdot 3600 \cdot 39.96} = 4237[Bq] \quad (17)$$

   We see that we have an natural activity of above 4kBq in our body originating from potassium-40. K-40 has the possibility to decay both by β^--decay (in 89% of the decays), and by electron capture decay to Ar-40, followed by the emission of a 1461 keV gamma (in 11% of the decays). We see that the beta-minus decay go directly to the ground state of Ca-40, with $Q_{\beta^-} = 1311$ keV. For the β^--decay we make the simple assmption that, on average, only half of the energy is left for the electron (the rest goes to the escaping anti-neutrino). With Katz and Penfold we see that the range of beta electrons below 1 MeV is just a few mm, so that almost all the beta-electrons are stopped inside the body. We therfore assume that approximately 650 keV ($Q_{\beta^-}/2$) of the beta-minus decay energy is deposited in the body. For the 1461 keV gamma, we have to assume the absorption in the body tissue. A rough estimate would be that an escaping gamma photon has to pass through about 10 cm of body tissue in order to escape the body. The ratio of gamma absorption then becomes:

$$\frac{I}{I_0} = e^{-\mu x} = e^{-5.701 \cdot 10^{-2} \cdot 1.06 \cdot 10} = 0.546, \quad (18)$$

where the $\mu\rho$-value for tissue at 1.5 MeV was used, together with a tissue density of 1.06 grams per cubic centimeter. For photons and betas over the whole body, the weighting factors are both 1. So, for the total effective dose D, we get:

$$D = \frac{A \cdot E_{tot} \cdot t}{m} = \quad (19)$$

$$\frac{4237 \cdot (650 \cdot 10^3 + 0.546 \cdot 1461 \cdot 10^3) \cdot 1.602 \cdot 10^{-19} \cdot 365 \cdot 24 \cdot 3600}{70} = \quad (20)$$

$$4.43 \cdot 10^{-4}[Sv] \quad (21)$$

We see that we get (from our own body) an effective dose of about 0.4 mSv over one year.

2. We must assume that we need a protected plate under the helicopter with a well defined area A. This area is independent of the material. Alpha and beta radiation form the ground is easily stopped, even by the air under the helicopter, so we can concentrate on only gamma radiation. We know that we want to minimise the expression e^{-mud}. For best absorption of gamma radiation, independent on energy, we therefore want to maximise the product $\mu \cdot d$ between the *linear absorption coefficient*, μ, (with unit cm^{-1}), and the thickness d (e.g. in unit cm). We have a well defined mass, m, of the protection plate as well, due to the limited lifting capacity. This means that we have a well defined *area density*, ρ_A, (e.g. with the unit g/cm^2). The thickness is then expressed (also using the volume density ρ): $d = \rho_A/\rho$. Now we see that the product we want to maximise is $\mu d = \mu\rho_A/\rho$. Since ρ_A is a constant in this problem, we should find the material with the highest $/mu/\rho$. This is in fact the *mass aborption coefficient* that is listed in the tables. So we simply take the material with the highest μ/ρ from the table. This material (using tha tables that were handed out) is *lead*. The activity at the time of the accident ($\mathcal{A}_0$)

is calculated like so: $mathcal A_0 = \lambda N$, where N is the number of nuclei in the sample. Assuming $N_{I133}/N_{Cs137} = 1/4$ we get for the ratio $\mathcal{R}$ of activities:

$$\mathcal{R} = \frac{\mathcal{A}_0(I133)}{\mathcal{A}_0(Cs137)} = \frac{1}{4}\frac{\lambda_{I133}}{\lambda_{Cs137}} = \quad (22)$$

$$\frac{1}{4}\frac{t_{1/2}(Cs137)}{t_{1/2}(I133)} = \frac{1}{4}\frac{30.17 \cdot 365.25 \cdot 24 \cdot 3600}{20.8 \cdot 3600} = \quad (23)$$

$$3179 \quad (24)$$

We see that, in the radiactive cloud, the activity from I^{133} is more that 3000 times higher than the activity from Cs^{137}. Here we have assumes that *four times more* in the problem refers to the number of nuclei, not the mass. Otherwise we should just multiply with the atomic mass ratio, and it will not change the result much, since 133/137 is close to 1. The

time until equal activities are reached is calculated by noting that the activity follow the exponential decay:

$$\mathcal{A} = \mathcal{A}_0 e^{-\lambda t} \quad (25)$$

We can now set the activites equal, and solve for the time:

$$\begin{aligned}
\mathcal{A}(I133) &= \mathcal{A}(Cs137) && (26)\\
\mathcal{A}_0(I133)e^{-\lambda(I133)t} &= \mathcal{A}_0(Cs137)e^{-\lambda(Cs137)t} && (27)\\
\frac{\mathcal{A}_0(I133)}{\mathcal{A}_0(Cs137)} &= \frac{e^{-\lambda(Cs137)t}}{e^{-\lambda(I133)t}} && (28)\\
\mathcal{R} &= e^{(\lambda(133)-\lambda(Cs137))\cdot t} && (29)\\
t &= \frac{ln(\mathcal{R})}{\lambda(I133)-\lambda(Cs137)} = 8.7125\cdot 10^5[s] && (30)
\end{aligned}$$

Dividing t with $24 * 3600$ gives us a time of around 10 days.

3. We can assume that all beta radiation is stopped easily by the water (Katz and Penfold will confirm this), so we perform the calculation only for gamma here. The Cs137 source will emit gamma radiation with the photon energy 662 keV. We can start by calculating the tranmission fraction in two metres of water and 20 cm of body tissue thickness. Here we use the absorption coefficients at 600 keV (and the tissue density) from the tables.

$$\mathcal{F}_{water} = e^{-0.08956\cdot 200} = 1.66\cdot 10^{-8} \quad (31)$$

$$\mathcal{F}_{body} = e^{-0.08873\cdot 1.06\cdot 20} = 0.15243 \quad (32)$$

Assuming a mass of 60 kg we get the dose:

$$\begin{aligned}
D &= \frac{\mathcal{F}_{water}(1-\mathcal{F}_{body})\mathcal{A}E_\gamma t}{m} = && (33)\\
&= \frac{1.66\cdot 10^-8(1-0.15243)10^3\cdot 3.7\cdot 10^{10}\cdot 662\cdot 10^3\cdot 1.602\cdot 10^{-19}\cdot 10\cdot 60}{60} && (34)\\
&= 0.553\cdot 10^{-6}[Sv] && (35)
\end{aligned}$$

We see that we get about 0.55 μSv, a very small dose. So there is no danger for the diver.

4. The ^{209}Bi-nuclei in the sample will absorb neutrons, due to the strong neutron flux in the reactor. Therefore a small amount of the Bi-209 nuclei will transform into Bi-210 nuclei. Assuming that it is formed in the ground state, it will decay by β^--decay to form the nuclide ^{210}Po. The half-life of this beta decay is about 5 days. The new nuclide ^{210}Po is unstable to alpha decay, with a half-life of 138 days. After one week, a little more than half of the Bi-210 will have decayed to Po-210, and it will continue to decay. At the same time, Po-210 will decay by alpha decay into Pb-206. All (100%) beta-decays from Bi-210 to Po-210 go directly to the gound state of Po-210, so there are no gamma emission from Po-210. *Almost* all of the alpha decays from Po-210 to Pb-206 go directly to the ground state of Pb-206, so we expect *almost* no gamma emission from Pb-206 either. There is a small fraction (0.00122%) of this alpha decay populating the 803 keV level in Pb-206, so we do expect a very weak gamma radiation. To summarise, after one week we expect almost no gamma photons (just a small intensity of 803 keV photons emitted from Pb-206). We also should see the beta decay electrons from the decay from Bi-210 to Po-210. This

should be a continous energy spectrum, with a maximum electron energy at the Q-value of 1163 keV. In addition, we expect to see alpha particle, emitted from the Po-210 nuclei. The alpha particles should have an energy of 5.304 MeV (from table of nuclides). Note that this is a lower energy than the alpha Q-value (5.407 MeV), due to the recoil energy of the Pb-206 nucleus. After 10 years, all the Bi-210 will have decayed to Po-210,

and almost all the Po-210 will have decayed to Pb-206 (it is easy to verify that only a fraction of close to 10^{-8} is left after 10 years.). This means that, in the chemical analysis of the sample after 10 years, we expect to see two elements; Bi-209 (some (most) nuclei did not absorb a neutron) and some amount of the stable lead isotope Pb-206.