Solutions to the Written Exam, Radiation Protection, Dosimetry, and Detectors (SH2603), Feb. 10, 2009, KTH, Stockholm

Section A

1. The daughter nucleus of Po210 (after the alpha decay) is Pb206. The daughter nucleus will get a *recoil* energy. One simple way to get the recoil energy is to use the difference between the total decay energy (i.e. the Q_α-value, from the table of isotopes, 5407.46 keV) and the alpha energy (from the nuclide chart 5.30438 keV). The difference is **103.2 keV**. This energy is the recoil energy of the daughter nucleus (Pb206). As an alternative, we can calculate the recoil energy by using the conservation of momenum as a starting point:

$$p(\alpha) = p(Pb206) \tag{1}$$
$$m(\alpha) \cdot v_\alpha = m(Pb206) \cdot v(Pb206) \tag{2}$$
$$v(Pb206) = \frac{m(\alpha)}{m(Pb206)} v(\alpha) \tag{3}$$
$$E(Pb206) = \frac{m(Pb206)v(Pb206)^2}{2} = \tag{4}$$
$$\frac{m(\alpha)^2}{m(Pb206)} \frac{v(\alpha)^2}{2} = \tag{5}$$
$$\frac{m(\alpha)}{m(Pb206)} E(\alpha) \tag{6}$$
$$Q_\alpha = E(Pb206) + E(\alpha); \tag{7}$$
$$E(Pb206) = \frac{Q_\alpha}{1+m(pb206)/m(\alpha)} \approx \frac{5407.46}{1+206/4} = 103\ [keV] \tag{8}$$

   We see that we get the same result. The recoil energy is 103 [keV].

2. We have, for the number, N, of nuclei, and the time, t:

$$N = N_0 e^{-\lambda t} \tag{9}$$
$$t = -\frac{t_{1/2} ln(0.43)}{ln(2)} = 6977\ [y], \tag{10}$$

   where we have used the ratio: $N/N_0 = 43/100 = 0.43$, and the half-life, $t_{1/2}$, of C-14 (5730 [y]). We see that the tool is about 7000 years old.

3. The 1 MeV gamma photon can interact with the NaI scintillator in (mainly) two ways; photo effect, and Compton scattering. In both cases, an electron within the NaI crystal will get a high kinetic energy (much higher than the binding energy of the electrons). The electron will distribute its energy to other electrons by Coulomb interaction. This will result in atomic excitations in the NaI crystal. (In the specific case of NaI, so-called *exitons*, i.e. electon-hole pairs, will be formed, travelling over the crystal until they find an impurity or dopant.) When the excited atomic structure is de-excited (by electrons falling into levels of lower energy), **light** will be emitted. If the matrial is transparent, this light will escape the scintillator crystal. Many light photons will be emitted for each detected gamma photon.

In order to transform the light into an electric pulse, we need some kind of converted outside of the scintillator. A common method is to use a *photo-multiplier*. The incoming light photon transfers its energy to an electron (photo-electric effect), the electron is accelerated in an electric field, hits a metalic plate and releases a number of electrons. The released electrons are then accelerated in a series of steps (typically about 10), until a strong enough electric (charge) puls is created.

Other converters is also used, e.g. PIN-diods.

4. The effective dose, H_E, is written like this:

$$H_E = \sum_T W_T \cdot H_T, \tag{11}$$

where W_T is the tissue weighting factor for the tissue T and H_T is the dose equivalent for the tissue T. Here, we have only one tissue/body part, i.e. the brain, and we can get $W_T = 0.05$ from a table. To calculate the dose equivalent, H, we use:

$$H = \sum_R W_R \cdot D_R, \tag{12}$$

where W_R is the radiation weighting factor. In this case we have only neutrons, so there is only one term in the sum. In this case $W_R = 5$, (from table) since the neutrons have energy lower than 10 keV. D_R is the absorbed dose. To get it, we need to know how much a brain weighs, and the table give us the value 1.5 kg. We now get the absorbed dose as:

$$D = \frac{2.5\ [J]}{1.5\ [kg]} = 1.667\ [Gy]. \tag{13}$$

The effective dose is now:

$$H_E = W_T \cdot W_R \cdot D = 0.05 \cdot 5 \cdot 1.667 \approx 0.417\ [Sv] \tag{14}$$

We see that the effective dose is quite high.

5. 50 mSv.

6. From the table of isotopes we see that the only gamma energy of interest is 662 keV, and that it is emitted in 94.4% of the decays. A source of 10 mCi emits gamma rays at a rate, R, of $0.944 \cdot 10 \cdot 10^{-3} \cdot 3.7 \cdot 10^{10} = 3.48 \cdot 10^8$ [s^{-1}]. The effective thickess, d, of the detector is 6 cm, and the solid angle fraction, f_A is:

$$f_A = \frac{\pi 3^2}{4\pi 40^2} = \frac{9}{6400} = 1.4 \cdot 10^{-3}. \tag{15}$$

Using the values (for germanium) of μ/ρ (using the value at 600 keV), and ρ from the NIST table, we get, for the total number of counts, T, per second in the detector:

$$T = R \cdot f_A \cdot (1 - e^{-\mu x}) = 3.48 \cdot 10^8 \cdot 1.4 \cdot 10^{-3} \cdot (1 - e^{-0.07452 \cdot 5.323 \cdot 6}) = 4.4 \cdot 10^5. \tag{16}$$

7. By neutron absorption, ^{244}Pu will first be transformed into ^{245}Pu. By β^--decay in two steps, the alpha-emitting nuclide ^{245}Cm (halflife 8500 years) is created.
First beta-decay (10.5 hours halflife): from ^{245}Pu to ^{245}Am.
Second beta-decay (2.05 hours halflife): from ^{245}Am to ^{245}Cm.
In both beta decays, several excited states are populated, resulting in radiation from gamma decay and conversion electrons.

8. The light nucleus ^{16}O has a density of approximately $0.2 \cdot 10^{18}\ kg/m^3$. What is the approximate density of the heavy nucleus ^{235}U?

   Nuclear density is approximately *constant* over the whole range The experimentally verified formula for the radius of the nucleus can be used to show this:

$$R = R_0 A^{1/3} \tag{17}$$
$$V = cR^3 = cR_0^3 A \tag{18}$$
$$\rho = \frac{M}{V} = \frac{kA}{cR_0^3 A} = \frac{k}{cR_0^3} = b, \tag{19}$$

   where R_0, k, c, and b, are constants. We see that the density is constant. Therefore, ^{235}U has the same density as ^{16}O.

   There are exceptions to this rule. Experiments measuring the density of some very neutron-rich nuclei (so-called halo nuclei) have revealed a much lower density than expected from the constant-density rule above.

9. From the NIST-tables, we get, for the Standard Nuclear emulsion; $\rho = 3.815[g/cm^3]$, and $\mu/\rho(60\ keV) = 3.693[cm^2/g]$. We now get:

$$I = I_0 e^{-\mu x} \tag{20}$$
$$\frac{1}{2} = e^{-\mu x} \tag{21}$$
$$ln(2) = \mu x \tag{22}$$
$$x = \frac{ln(2)}{\mu} = \frac{ln(2)}{3.815 \cdot 3.693} = 0.0492[cm]. \tag{23}$$

   We see that approximately 0.5 mm is suitable for the film thickness.

10. From the nuclide chart, we get the present abundances and half-lives for U-238 and U-235. We now get:

$$N = N_0 e^{-ln(2)t/t_{1/2}} \tag{24}$$
$$\frac{N_{235}}{N_{238}} = \frac{N_{0,235} e^{-ln(2)t/t_{1/2}(U235)}}{N_{0,238} e^{-ln(2)t/t_{1/2}(U238)}} \tag{25}$$
$$\frac{0.007204}{0.992742} = e^{-ln(2)t(1/t_{1/2}(U238) - 1/t_{1/2}(U235))} \tag{26}$$
$$ln\left(\frac{0.007204}{0.992742}\right) = ln(2)t\left(\frac{1}{t_{1/2}(U238)} - \frac{1}{t_{1/2}(U235)}\right) \tag{27}$$
$$t = \frac{1}{ln(2)} \cdot ln\left(\frac{0.007204}{0.992742}\right) \cdot \frac{1}{\left(\frac{1}{t_{1/2}(U238)} - \frac{1}{t_{1/2}(U235)}\right)} = \tag{28}$$

$$= \frac{1}{ln(2)} \cdot ln\left(\frac{0.007204}{0.992742}\right) \cdot \frac{1}{\left(\frac{1}{4.468 \cdot 10^9 [y]} - \frac{1}{7.04 \cdot 10^8 [y]}\right)} = \tag{29}$$

$$= 5.94 \cdot 10^9 \ [y]. \tag{30}$$

where $N_{0,235}$ and $N_{0,238}$ are the (assumed equal) abundances of the two isotopes at the time of Earth's creation, and wher N_{238} N_{235} are the two present abundances. We see that, with this simplified approach, we get an age for the Earth of about $6 \cdot 10^9$ years. More detailed studies (using various radioisotopes) find the age to be close to $4.5 \cdot 10^9$ years.

Section B

1. We have the following relation between number and activity:

$$A = -\frac{dN}{dt} = \lambda \cdot N = \frac{N \cdot ln(2)}{t_{1/2}} \quad (31)$$

$$N = \frac{A \cdot t_{1/2}}{ln(2)} \quad (32)$$

Here, we have, per kg:

$$N_{liver} = \frac{A \cdot t_{1/2}}{ln(2)} = \frac{0.55 \cdot 10^{-12} \cdot 3.7 \cdot 10^{10} \cdot 2.4 \cdot 10^4 \cdot 3600 \cdot 24 \cdot 365}{ln(2)} = 2.2 \cdot 10^{10} \quad (33)$$

$$N_{skeleton} = \frac{A \cdot t_{1/2}}{ln(2)} = \frac{0.22 \cdot 10^{-12} \cdot 3.7 \cdot 10^{10} \cdot 2.4 \cdot 10^4 \cdot 3600 \cdot 24 \cdot 365}{ln(2)} = 8.9 \cdot 10^9 \quad (34)$$

With a skeleton mass of 10 kg, and a liver mass of 1.5 kg, we get the total number, N, of Pu239 atoms in the body:

$$N = 2.2 \cdot 10^{10} \cdot 1.5 + 8.9 \cdot 10^9 \cdot 10 = \quad (35)$$

$$= (3.3 \cdot 10^{10})_{liver} + (8.9 \cdot 10^{10})_{skeleton} = \quad (36)$$

$$= (1.2 \cdot 10^{11})_{total} \quad (37)$$

The total number of Pu239 atoms in the body is $1.2 \cdot 10^{11}$.

We now calculate the dose for each organ, per year. Here, we can make the approximation that only alpha particles contribute to the dose, since the gamma photons have much lower energies (mainly 77 keV, as compared to the Q_α-value of 5.244 MeV), and have lower W_R. We can take the full $Q - value$ since the recoiling daughter nucleus will also contribute to the dose. We now get the absorbed dose by using the activity per kg for the two organs:

$$D_{liver} = \frac{AEt}{m} = \quad (38)$$

$$= 0.55 \cdot 10^{-12} \cdot 3.7 \cdot 10^{10} \cdot 5.244 \cdot 10^6 \cdot 1.602 \cdot 10^{-19} \cdot 3600 \cdot 24 \cdot 365 \quad (39)$$

$$= 5.39 \cdot 10^{-7} \ [Gy] \quad (40)$$

$$D_{skel} = \frac{AEt}{m} = \quad (41)$$

$$= 0.22 \cdot 10^{-12} \cdot 3.7 \cdot 10^{10} \cdot 5.244 \cdot 10^6 \cdot 1.602 \cdot 10^{-19} \cdot 3600 \cdot 24 \cdot 365 \quad (42)$$

$$= 2.16 \cdot 10^{-7} \ [Gy] \quad (43)$$

The effective dose is found by muliplying the absorbed dose with the tissue weighting factor, W_T for each organ, and then summing up the organs, and finally by muliplying with the radiation weighting factor, W_R. Here, W_R is 20. W_T is 0.05 for the liver. For the skeleton, it might not be obvious which value in the table to use, but for this solution we use the value of 0.01 (bone surface, e.g. disregarding the bone marrow. We now have, for the effective dose, H:

$$H = W_R \cdot (D_{liver} \cdot W_T(liver) + D_{skeleton} \cdot W_T(bone) = \quad (44)$$

$$20 \cdot (5.39 \cdot 10^{-7} \cdot 0.05 + 2.16 \cdot 10^{-7} \cdot 0.01) = 5.8 \cdot 10^{-7} \ [Sv] \quad (45)$$

The effective dose is about 0.6μSv per year, i.e. a very small fraction of the average dose received from other sources (about 4 mSv).

2. Since the child suffers from radiation sickness, but still survives, we can assume that the effective dose is around 1-2 Sv (this is the answer to the first part of the problem). For the rest of the calculation, we assume that the received effective dose is 1 Sv.

   The nuclide Am241 decays by alpha emission, but the alpha particles will be completely stopped by the plastic around the source. The daughter nucleus (Np237) is however populated in several excited states that decay by gamma emission. If we make the simplified assumption that all gamma rays penetrate the plastic surrounding the source, we need only to consider the absorption in the body tissue. The most intense gamma line in this decay is 59.5 keV, but there are also other gamma energies. We consider, for now, two populated states in the daughter nucleus Np237; the 59.5 keV state (85.2% of decays) and the 103.0 keV state (12.8% of decays). For the intensity (branching ratio) of the different gamma energies, we get:

$$59.5\ [keV]: \quad 0.852 \cdot \tfrac{100}{100+6.71} + 0.128 \cdot \tfrac{100}{100+26.7+4} \cdot \tfrac{100}{100+6.71} = 0.89 \tag{46}$$

$$43.4\ [keV]: \quad 0.128 \tfrac{100}{100+26.7+4} = 0.097 \tag{47}$$

$$26.3\ [keV]: \quad 0.852 \cdot \tfrac{6.71}{100+6.71} + 0.128 \cdot \tfrac{100}{100+26.7+4} \cdot \tfrac{6.71}{100+6.71} = 0.060 \tag{48}$$

$$33.2\ [keV]: \quad 0.852 \cdot \tfrac{6.71}{100+6.71} + 0.128 \cdot \tfrac{100}{100+26.7+4} \cdot \tfrac{6.71}{100+6.71} + \tag{49}$$

$$+0.128 \tfrac{4}{100+26.7+4} = 0.064 \tag{50}$$

$$69.8\ [keV]: \quad 0.128 \tfrac{4}{100+26.7+4} = 0.0039 \tag{51}$$

$$103.0\ [keV]: \quad 0.128 \tfrac{26.7}{100+26.7+4} = 0.026 \tag{52}$$

   We continue by considering the absorption of this gamma energy in the child's body. Assuming a thickness of tissue (from the source to the body surface) of 10 cm, we get the following absorption ratios (I/I_0), for the various energies involved:

$$59.5\ [keV]: \frac{I}{I_0} = e^{-\mu x} = e^{-(\mu/rho)rhox} = e^{-0.2048 \cdot 1.06 \cdot 10} = 0.11 \tag{53}$$

$$42.7\ [keV]: \frac{I}{I_0} = e^{-\mu x} = e^{-(\mu/rho)rhox} = e^{-0.2688 \cdot 1.06 \cdot 10} = 0.058 \tag{54}$$

$$26.3\ [keV]: \frac{I}{I_0} = e^{-\mu x} = e^{-(\mu/rho)rhox} = e^{-0.6 \cdot 1.06 \cdot 10} = 0.0017 \tag{55}$$

$$33.2\ [keV]: \frac{I}{I_0} = e^{-\mu x} = e^{-(\mu/rho)rhox} = e^{-0.379 \cdot 1.06 \cdot 10} = 0.018 \tag{56}$$

$$69.8\ [keV]: \frac{I}{I_0} = e^{-\mu x} = e^{-(\mu/rho)rhox} = e^{-0.19 \cdot 1.06 \cdot 10} = 0.13 \tag{57}$$

$$103.0\ [keV]: \frac{I}{I_0} = e^{-\mu x} = e^{-(\mu/rho)rhox} = e^{-0.1693 \cdot 1.06 \cdot 10} = 0.17 \tag{58}$$

$$\tag{59}$$

where we have used the absorption coefficients for energies *near* the specified ones. Multiplying the energy with the branching ratio, R_b and with the absorption ratio we get the average energy deposited (per decay):

$$59.5\ [keV] : (1 - 0.11) \cdot 0.89 \cdot 59.5 = 47.1\ [keV] \quad (60)$$
$$42.7\ [keV] : (1 - 0.058) \cdot 0.097 \cdot 42.7 = 3.9\ [keV] \quad (61)$$
$$26.3\ [keV] : (1 - 0.0017) \cdot 0.060 \cdot 26.3 = 1.5\ [keV] \quad (62)$$
$$33.2\ [keV] : (1 - 0.018) \cdot 0.064 \cdot 33.2 = 2.1\ [keV] \quad (63)$$
$$69.8\ [keV] : (1 - 0.13) \cdot 0.0039 \cdot 69.8 = 2.4\ [keV] \quad (64)$$
$$103.0\ [keV] : (1 - 0.17) \cdot 0.026 \cdot 103.0 = 2.2\ [keV] \quad (65)$$

The sum of these energies is 59 keV. We see now that we could perhaps have made the approximation to use only the 59.5 keV gamma, this would have given an error of about 20% (and our 10 cm assumtion will most likely give a bigger error). With the calcuation above, we see that the total energy deposited (by gamma) in the body, is 59 keV, for each decay. Now we can calculate the activity. We know that the radiation weighting factor for photons is 1, and we assume full-body exposure. Then, the effective dose, D, can be written as:

$$D = \frac{A \cdot E \cdot t}{m} \quad (66)$$

where A is the activity, E is the deposited energy per decay, t is the time, and m is the mass. We now get the activity:

$$A = \frac{D \cdot m}{E \cdot t} = \frac{1 \cdot 25}{59 \cdot 10^3 \cdot 1.602 \cdot 10^{-19} \cdot 3600 \cdot 24} = 3.06 \cdot 10^{10}\ [Bq] = 0.83\ Ci. \quad (67)$$

With the assumption of an effective dose of 1 Sv, the activity of the source was 0.83 Ci. It is worth noting that the child would surely have died if this would have been an *open* source at the same activity, since the high alpha energy, and the weighting factor for alpha together would have increased the dose by more than a factor of 1000.

3. The radiactive krypton isotope of concern is ^{85}Kr (all other neutron rich krypton isotopes had decayed to very small amounts by the time of the gas release). Only the decay from the ground state of Kr85 should be considered. Kr85 decays by beta minus, i.e. emitting beta electrons. Only a very small percentage (less than 0.5%) populates an excited state in Rb85. The main concern is therefore the *beta electrons*, with continous electron energies from zero up to the Q-value 687 keV. If the gas was inhaled, the electrons would then deposit their full energy inside the body of the person.

   The activity concentration c_A would be:

$$c_A = \frac{45000}{(10^3)^3} = 4.5 \cdot 10^{-5}\ [Ci/m^3] = 4.5 \cdot 10^{-8}\ [Ci/dm^3] = 1665\ [Bq] \quad (68)$$

   If we set the average electron energy to half the Q-value, i.e 344 keV, and assume that the average lung volume is 3 dm^3, we have, for 24 hours, a dose D:

$$D = \frac{AEt}{m} = \frac{1665 \cdot 344 \cdot 10^3 \cdot 1.602 \cdot 10^{-19} \cdot 3600 \cdot 24}{70} = 1.13 \cdot 10^{-7} \ [Gy] \quad (69)$$

for a person of 70 kg body mass. Electrons ($W_R = 1$), and full body dose, gives us an effective dose of 0.11 μSv. We note that the volume of gas dilution (here 10^9 cubic metres), is very important for the result.

4. In this problem, we can rather easily neglect several sources, since they will not contribute much to the final dose. For the activity we have

$$A = A_0 e^{-tln(2)/t_{1/2}}, \quad (70)$$

where A_0 is the original activity. Co60 has a half-life of 5.27 years. The activity was measured 46 years ago. The activity today is then about 0.0024 times the original activity, e.f. 0.024 mCi. Co57 has a short half-life of 272 days, and will have only a fraction of 0.00058 of the original activity left. For Y88, the fraction left is below 10^{-7}. All the above can therefore be neglected directly. Sr90 is an open source, but emits (almost) only beta electrons (and anti-neutrinos). The beta electrons will be completely stopped by the iron box. The only two sources left to consider is Am241, and Cs137. The alpha from Am241 will be stopped, since it is a closed source. The Am241 source has, first of all, a much lower activity than the Cs137 source. But, in addition, the strongest gamma line in Am241 is only about 60 keV. This contributes very little to the final dose, not only due to the low energy, but because the low energy gamma photons will be rather effectively stopped by the iron box. So, according to the above reasonling, we need only to consider Cs137. The beta electrons will be stopped within the box, but the gamma photons will escape. The present activity of the Cs137 source is:

$$A = A_0 e^{-tln(2)/t_{1/2}} = 10 \cdot 10^{-3} \cdot 3.7 \cdot 10^{10} \cdot e^{-ln(2)37/30.17} = 1.58 \cdot 10^8 \ [Bq]. \quad (71)$$

The problem is now reduced to calculating the absorption fraction, f_{box}, in the iron ($d_1 = 0.2 \ cm$), the absorption in the lab assistant ,f_a, (assumed average thicknes $d_2 = 25 \ cm$), and the solid angle, f_s. The absorption coefficients should be taken near the gamma energy of the Cs137 source, i.e. 662 keV.

$$f_{box} = \frac{I}{I_0} = e^{-\mu x} = e^{-0.077 \cdot 7.84 \cdot 0.2} = 0.89 \quad (72)$$

$$f_a = \frac{I}{I_0} = e^{-\mu x} = e^{-0.089 \cdot 1.06 \cdot 25} = 0.095 \quad (73)$$

$$f_s = \frac{1.75 \cdot 0.35}{4\pi(8^2 + 5^2)} = 5.48 \cdot 10^{-4} \quad (74)$$

where we have assumed a 1.75 m tall and 0.35 m wide assistant. We can now get the dose, D:

$$D = \frac{AtEf_{box}(1 - f_a)f_s}{m} = ... = 3.0 \cdot 10^{-6} \ [Gy]. \quad (75)$$

where a mass of 70 kg is assumed for the lab assistant. We have not considered the angle dependence on the thickness of the iron box but used the 2 mm value directly. (Note! Since the absorption coefficient for iron was not given in tha tables, the value for another metal can be used. If the correct density is used, this will not affect the result much, since the abs. coeff in the table is normalised for density). We have photons, and full body exposure, so the final effective dose is: 3 [μSv]. to reduce the dose by a factor of 10, we can (for example) put lead plates on the inner or outer walls of the box. We have for the thickness, d_{Pb}:

$$\frac{I}{I_0} = 0.1 = e^{-11.34 \cdot 0.1248 \cdot d_{Pb}} \tag{76}$$

$$d_{Pb} = \frac{-ln(0.1)}{0.1248 \cdot 11.34} = 1.63\ [cm] \tag{77}$$

We see that lead walls with a thickness of 1.63 cm will reduce the dose by a factor of 10. They can be attached on the outer or inner (if there is space available) walls of the box.