Homeworkproblems 2

Exercise 1:
Show that under a parity transformation a particle with angular momentum l and total spin j carries a parity $\pi = (-1)^l$.

Exercise 2: a) Find the eigenvalues and eigenvectors of the impulse operator in one dimension, i. e. $p_x\varphi(x) = k\varphi(x)$, where $p_x = \frac{\hbar}{i}\frac{d}{dx}$, within the range $0 \leq x \leq L$ and the boundary condition $\varphi(0) = \varphi(L)$.
b) The same for p_x^2 with $\varphi(0) = \varphi(L) = 0$.

Exercise 3: Write the unit operator (projector) corresponding to the Hilbert space spanned by the representation given by the eigenfunctions $\varphi(x)$ of the one-particle Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x),$$

where $V(x) = \infty$ for $x = 0$, $V(x) = -V_0$ for $0 < x \leq a$ and $V(x) = 0$ for $x > a$. Assume that the wave function in the continuum is normalized to unity within the region $0 < x \leq L$.

Exercise 4:
Evaluate the magnetic dipole moment (Schmidt value) corresponding to
a) A proton moving in an orbital $l = 2, j = 5/2$.
b) A neutron moving in an orbital $l = 1, j = 1/2$.

Exercise 5:
a) The magnetic dipole moments corresponding to the ground states of the nuclei $^{17}F_8$ and $^{41}Sc_{20}$ are observed to be 4.72 and 5.93, respectively. Which are the corresponding values of l and j?.
b) Which values of l and j would have the ground states of $^{17}O_9$ and $^{41}Ca_{21}$, for which the magnetic dipole moments are measured to be -1.83 and -1.59, respectively?.