Homeworkproblems 1

Exercise 1: Using the eigenvectors and eigenvalues of the Pauli matrix σ_z as a basis evaluate the eigenvalues and eigenvectors of σ_x and σ_y, where $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Exercise 2:
a) Why translating the system of coordinates by an amount $+\mathbf{\Delta r}$ the function $\Psi(\mathbf{r})$ becomes $\Psi(\mathbf{r}-\mathbf{\Delta r})$? (Eq. (8) of Chapter 1).
b) Show that the parity operator is Hermitian.

Exercise 3:
Show that if the operators A and B conmute, then they have common eigenvectors.

Exercise 4:
a) Show that $|j_1 j_2 JM >= (-1)^{j_1+j_2-J} |j_2 j_1 JM >$
b) Evaluate the following Clebsh-Gordan coefficients:
i) $< j{+}1/2\; j{-}1/2\; 1/2\; 1/2 | j\; j >$, ii) $< j{+}1/2\; j{+}1/2\; 1/2{-}1/2 | j\; j >$. iii) $< j{-}1/2\; j{-}1/2\; 1/2\; 1/2 | j\; j >$ and iv) $< j-1/2\; j+1/2\; 1/2\; -1/2 | j\; j >$.

Exercise 5:
a) Which is the relation between m_1, m_2 and m, and between j_1, j_2 and j in the Clebsh-Gordon coefficient $CG =< j_1 m_1 j_2 m_2 | jm >$.
b) Show that $CG =< jmjm|JM >=0$ if $2j - J$ is odd. Which is the value of J if $j = 1/2$?.
c) Show that $\begin{Bmatrix} \frac{9}{2} & 1 & \frac{7}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} \end{Bmatrix} = 0$ and $\begin{Bmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{Bmatrix} = 0$.

Short Algebraic Table of the 3-j Symbols

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ 0 & 0 & 0 \end{pmatrix} = (-1)^{J/2} \left[\frac{(J-2j_1)!(J-2j_2)!(J-2j_3)!}{(J+1)!} \right]^{1/2} \frac{(\frac{1}{2}J)!}{(\frac{1}{2}J-j_1)!(\frac{1}{2}J-j_2)!(\frac{1}{2}J-j_3)!}$$

where $J = j_1 + j_2 + j_3$ is even.

$$\begin{pmatrix} j & j & 0 \\ m & -m & 0 \end{pmatrix} = (-1)^{j-m} \left[\frac{1}{2j+1} \right]^{1/2}$$

$$\begin{pmatrix} j+\frac{1}{2} & j & \frac{1}{2} \\ m & -m-\frac{1}{2} & \frac{1}{2} \end{pmatrix} = (-1)^{j-m-\frac{1}{2}} \left[\frac{j-m+\frac{1}{2}}{(2j+2)(2j+1)} \right]^{1/2}$$

$$\begin{pmatrix} j+1 & j & 1 \\ m & -m-1 & 1 \end{pmatrix} = (-1)^{j-m-1} \left[\frac{(j-m)(j-m+1)}{(2j+3)(2j+2)(2j+1)} \right]^{1/2}$$

$$\begin{pmatrix} j+1 & j & 1 \\ m & -m & 0 \end{pmatrix} = (-1)^{j-m-1} \left[\frac{2(j+m+1)(j-m+1)}{(2j+3)(2j+2)(2j+1)} \right]^{1/2}$$

$$\begin{pmatrix} j & j & 1 \\ m & -m-1 & 1 \end{pmatrix} = (-1)^{j-m} \left[\frac{2(j-m)(j+m+1)}{(2j+2)(2j+1)(2j)} \right]^{1/2}$$

$$\begin{pmatrix} j & j & 1 \\ m & -m & 0 \end{pmatrix} = (-1)^{j-m} \frac{2m}{[(2j+2)(2j+1)(2j)]^{1/2}}$$